

# QUANTUM PHASES OF MATTER IN HYPERBOLIC SPACE

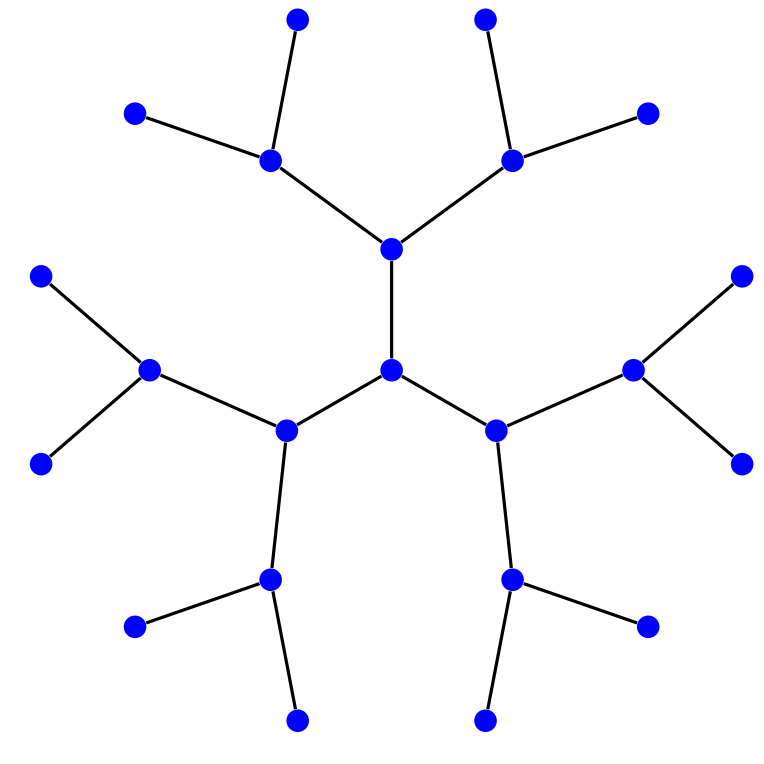
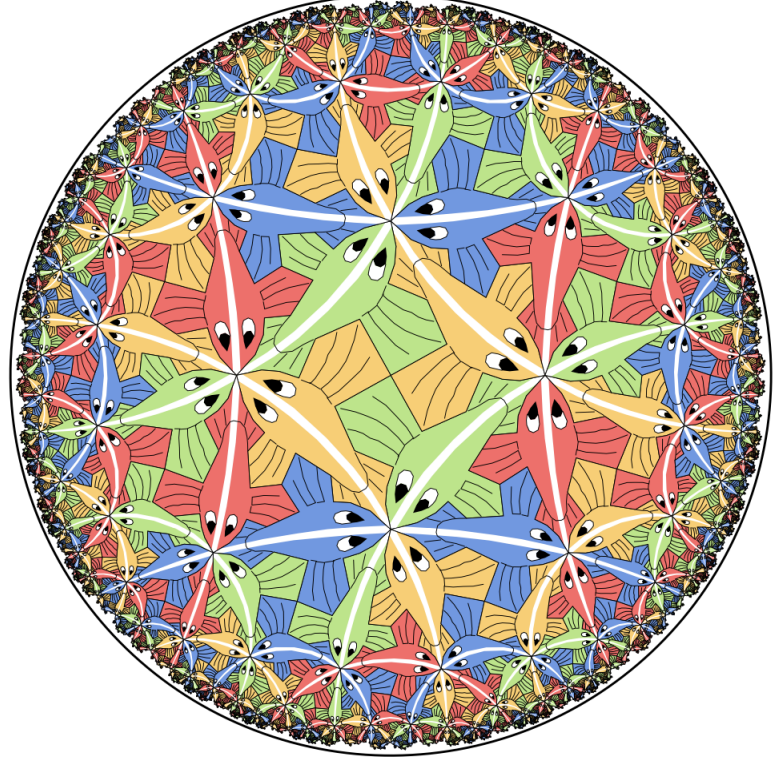
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## Spin Models on Hyperbolic Lattices

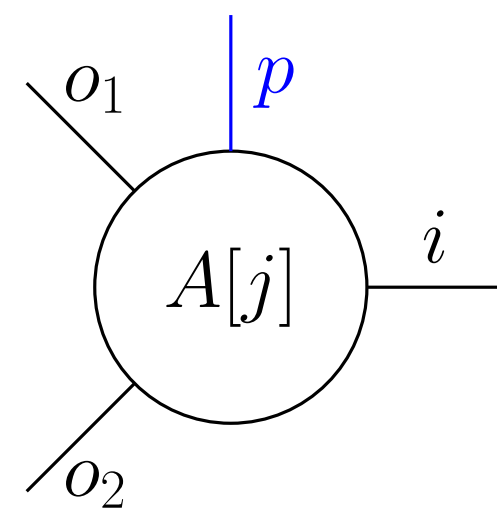


- Key Property: Size of boundary/interior are comparable (isoperimetric inequality).
- We study quantum spin models on the *Cayley trees* - finite  $z$ -regular graphs without loops, and their infinite-size counterpart, the *Bethe lattice* [1].
- Our investigations center on the quantum transverse-field Ising model:

$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_i X_i.$$

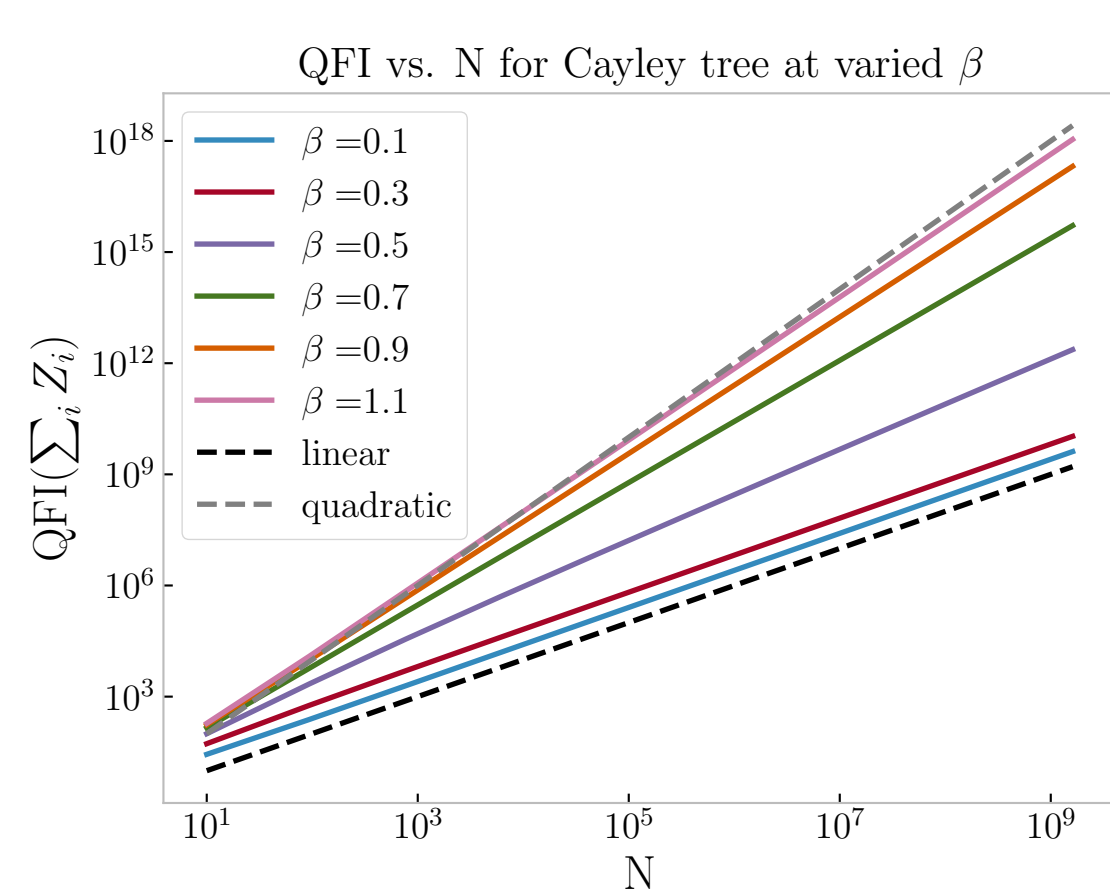
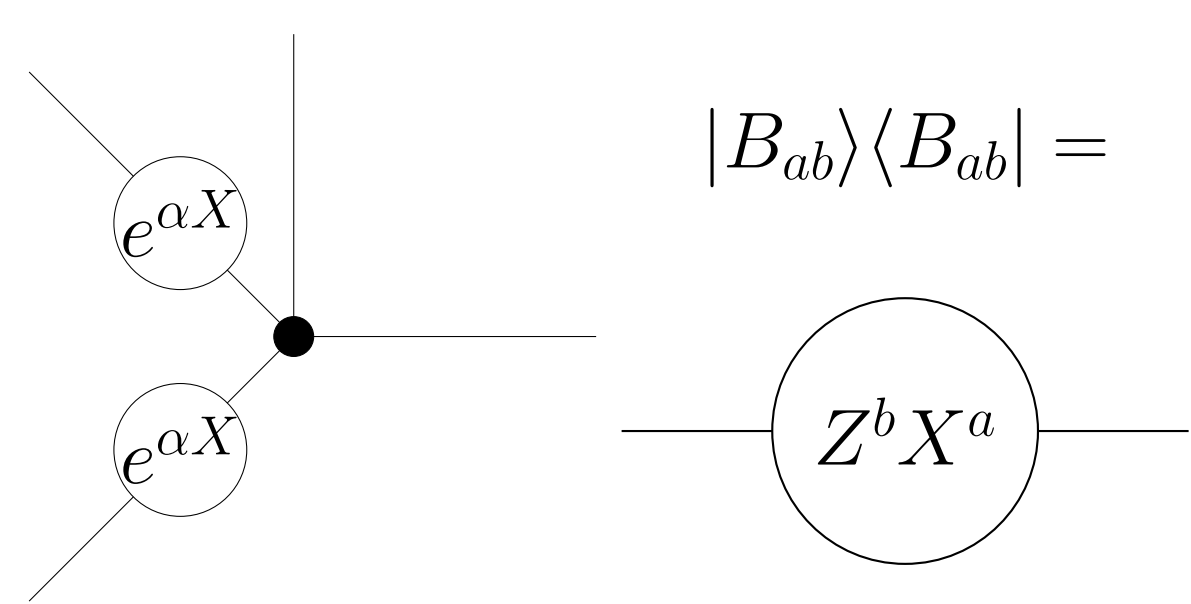
- Hyperbolic weirdness: Previous studies on the Bethe lattice show non-divergent correlation lengths at phase transitions [2] and an absence of Goldstone Bosons [3].

## Central Tool - Tensor Networks



- Rotational Invariance  $\implies$   $L$ -tensor representation ( $L$ -ring Cayley Tree)
- +Translation Invariance  $\implies$  single-tensor representation (Bethe Lattice)
- Real/Imaginary Time evolution  $e^{itH}/e^{\beta H}$  implemented via Trotterized MPO.
- Cavity-method [4] inspired technique of finding fixed point of spatial contractions to find energy spectrum.

## Measurement-Based Preparation of Critical States



- Adapting the technique of [5], we can prepare the “deformed Ising” state:

$$|\psi(\beta)\rangle = \exp\left(\sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$$

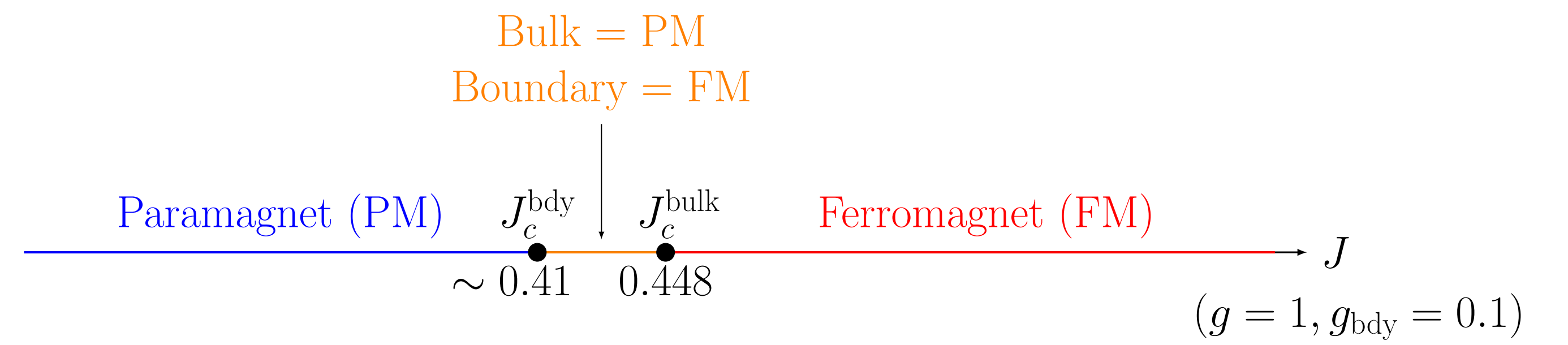
on the Cayley tree in finite time by (a) preparing many 4-qubit spiders (left), (b) Fusing spiders via Bell measurements (center) to form the tree, (c) Pushing through error/Pauli operators to the tree boundary, and (d) performing a single round of error cleanup.

- (Right) We can explicitly calculate the Quantum Fisher Information:

$$\text{QFI}_{\beta}\left(\sum_i Z_i\right) = \left\langle \sum_{ij} Z_i Z_j \right\rangle_{\beta} - \left\langle \sum_i Z_i \right\rangle_{\beta}^2,$$

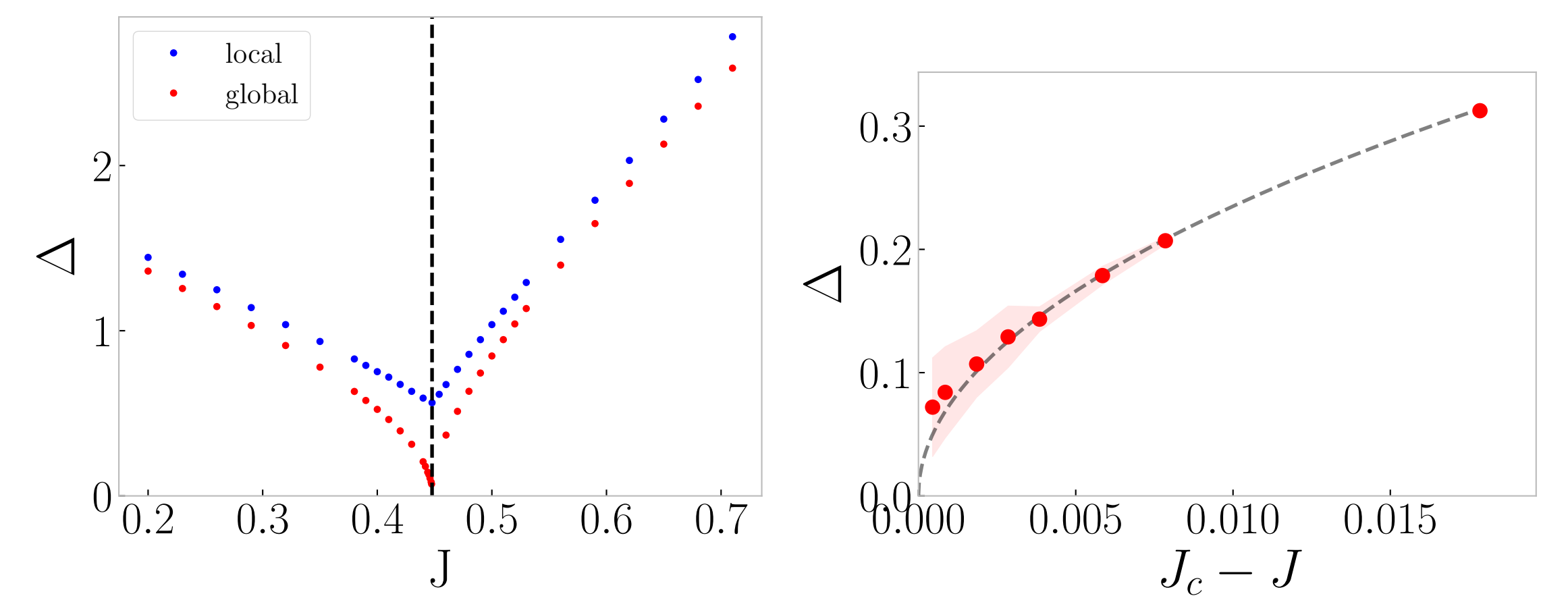
which signals a transition (linear  $\rightarrow$  quadratic) at  $\beta_c = \frac{1}{2} \ln\left(\frac{z}{z-2}\right) \approx 0.55$ .

## Phase Diagram



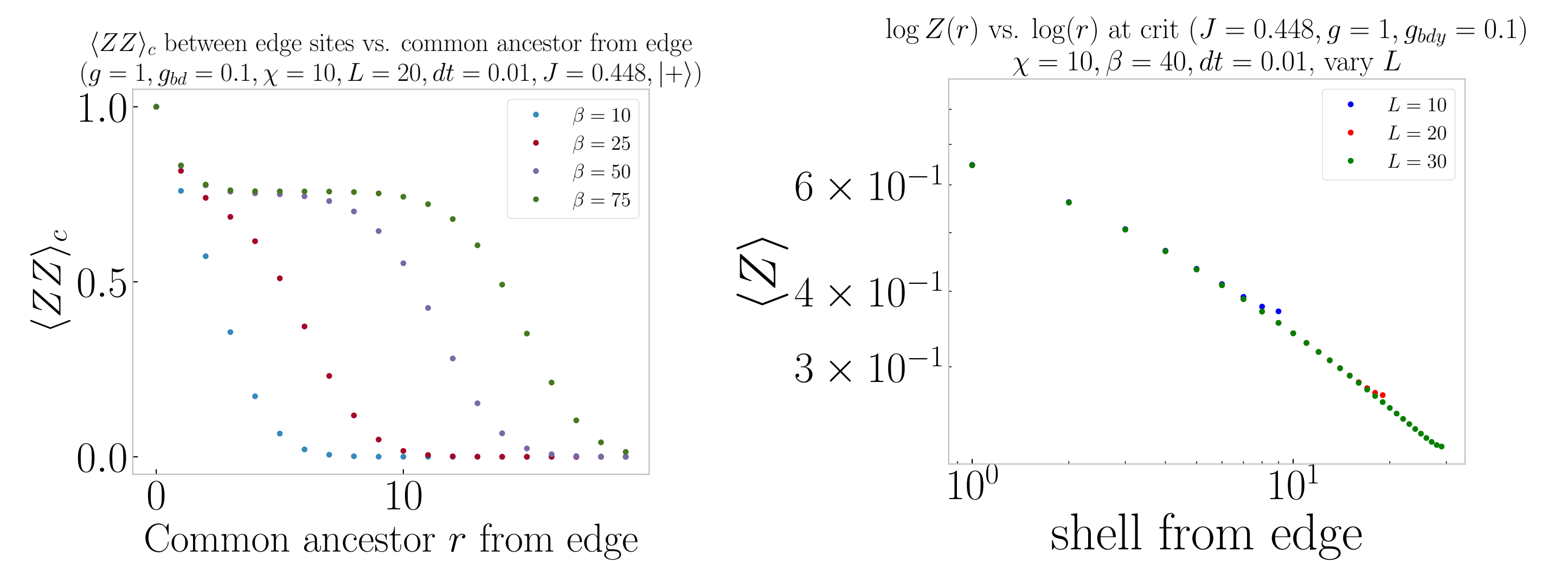
- Phases analyzed by looking at magnetization  $\langle Z \rangle$  at boundary/bulk of Cayley tree ground states (obtained by imaginary time evolution).
- Confirms the Bethe critical point  $J_c = 0.448$  of [2].
- Reveals an intermediate boundary ordered/bulk disordered phase - echoing phenomenology of the boundary-sensitive phase of the classical Ising model [6] and boundary-dominated superconducting phases for BCS models on hyperbolic trees [7].

## Energy Gaps



- Global gap vanishes ( $\sim \sqrt{J_c - J}$ ) at the phase transition. Local gap stays open!
- Supports observations of [3] of the quantum spherical model on the Bethe lattice, where a gap vanishes to global modes but local correlators remain gapped.

## Correlations



- (Left) At criticality, a cat state develops on the boundary of the Cayley tree as we cool into the ground state.
- (Right) At criticality, we see algebraic(!) correlations. Hastings-type bounds [8] of  $\Delta > 0 \implies$  exponential decay are evaded due to degeneracy at the boundary.
- Ground states for  $J < J_c$  exhibit correlations with  $\xi < (\ln 2)^{-1}$ , while those for  $J > J_c$  exhibit correlations with  $\xi > (\ln 2)^{-1}$ . Thus, a nonzero local gap opens the exciting possibility of preparing long-range correlated states in finite time with local (unitary) time evolution:

$$U(t) = \exp\left(i \int_0^t H(t') dt'\right)$$

by sweeping  $J$  across  $J_c$  (Work-in-progress!).

## References

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