Quantum Phases of Matter in Hyperbolic Space

APS Global Summit 2026

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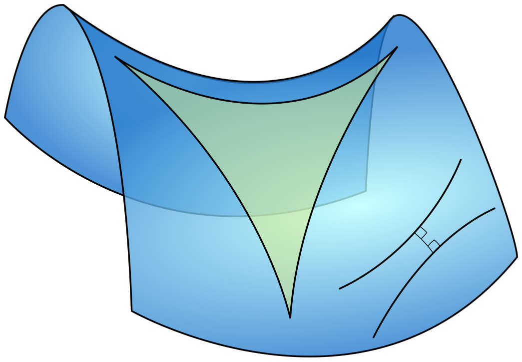
Background/Motivation - What is hyperbolic space?



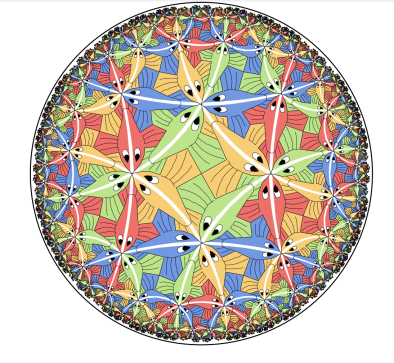
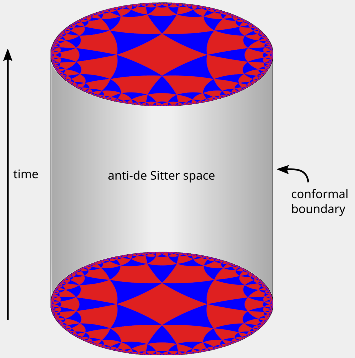
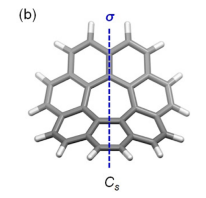
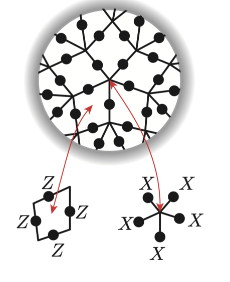


Image Credits: Wikipedia; D. Dunham (Transformation of Hyperbolic Escher Patterns)

Background/Motivation - Hyperbolic space is everywhere!







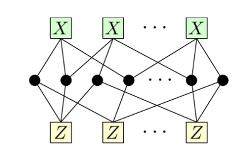
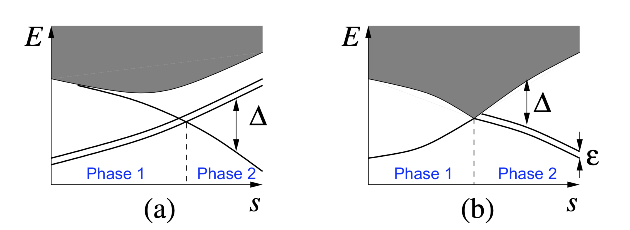
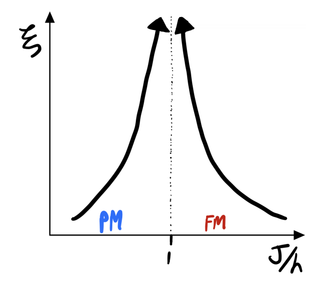


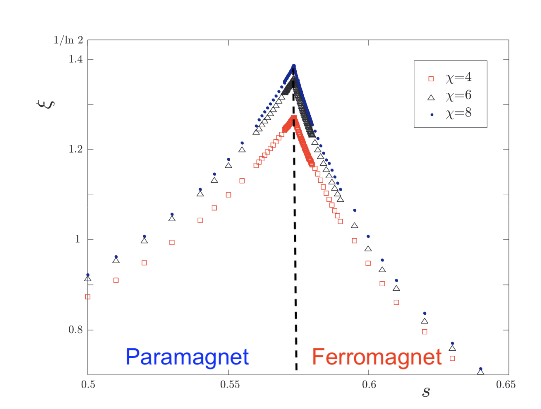
Image Credits: Wikipedia; Org. Lett. 2017, 19, 9, 2246-2249; arXiv:1703.00590; Quantum 5, 585 (2021)

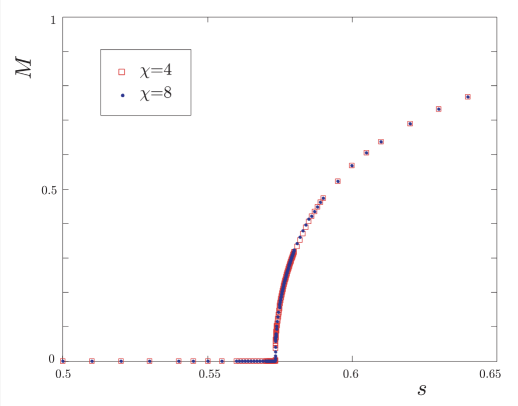




- Euclidean: Established relationships between adiabaticity/phases/gaps/correlations.

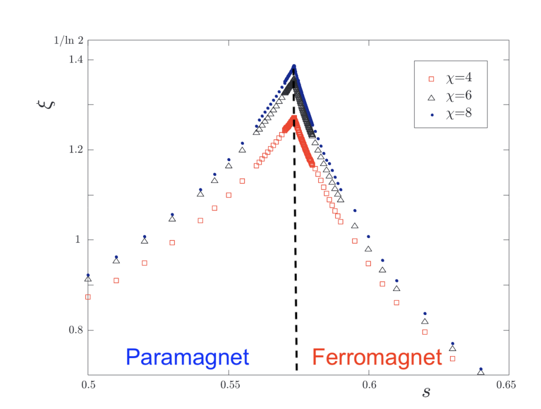
Image Credit: Phys. Rev. B 82, 155138

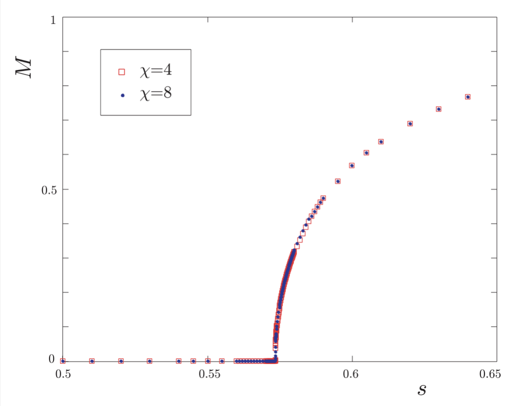




- Euclidean intuitions can be broken...
  - Non-divergent correlation length at phase transition *[Nagaj et. al, Phys. Rev. B (2008)]*
  - No Goldstone bosons *[Lauman et. al, Phys. Rev. B (2009)]*

Image Credit: Phys. Rev. B 77, 214431





- Euclidean intuitions can be broken...
  - Non-divergent correlation length at phase transition *[Nagaj et. al, Phys. Rev. B (2008)]*
  - No Goldstone bosons *[Lauman et. al, Phys. Rev. B (2009)]*
- Central Question: Does the energy gap need to close at the transition?

Image Credit: Phys. Rev. B 77, 214431

Setting - TFIM on Cayley Tree/Bethe Lattice

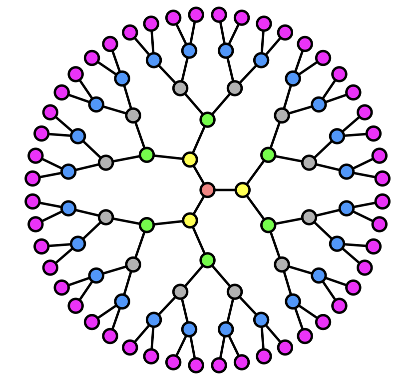


Image Credit: arXiv:1406.2819

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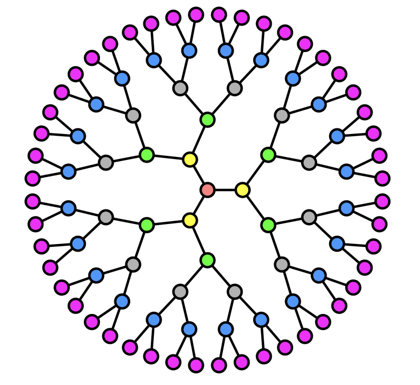
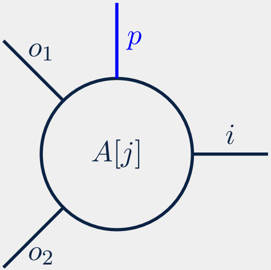
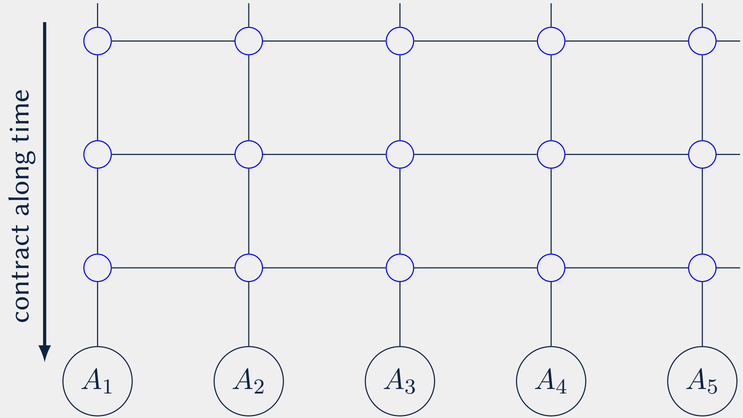


$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - \sum_i X_i$$

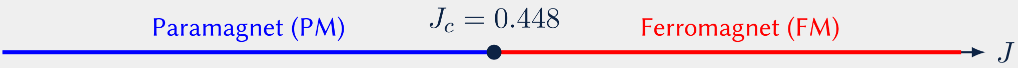
Image Credit: arXiv:1406.2819



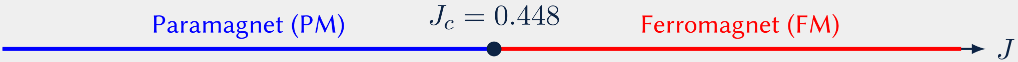
- Bethe - Store a single 4-legged tensor.
- Cayley - Rotational symmetry $\implies$ L tensors for L rings.

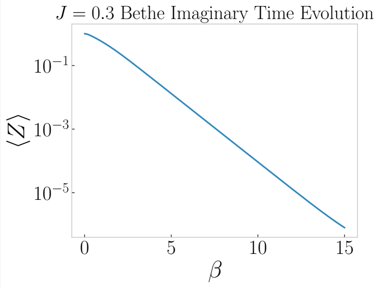


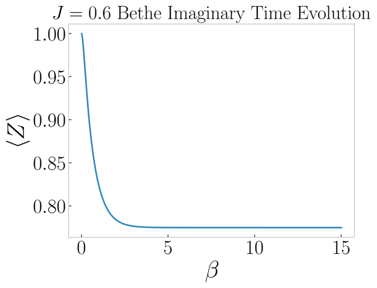
- Trotterized $\exp(\beta H)$ or $\exp(itH)$ for imaginary/real time evolution.


Paramagnet (PM)
$J_c = 0.448$
Ferromagnet (FM)
J

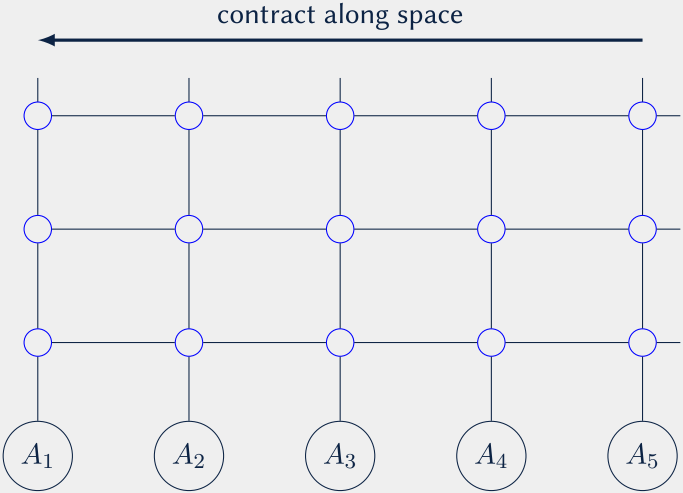
Results - Phase Diagram



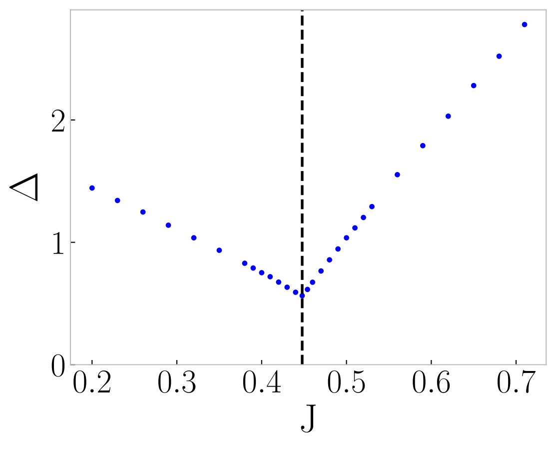




- Reproduces known bulk transition point of *[Nagaj et. al, Phys. Rev. B (2008)]*



- Transfer matrix eigenvalues of (spatial) fixed-point MPS yields a probe of the spectrum.

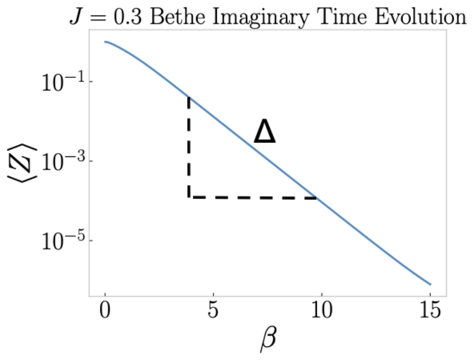


- Gap stays open at J_c!

$$\langle\uparrow| e^{-\beta H} Z e^{-\beta H} |\uparrow\rangle = \sum_{nm} e^{-\beta(E_n+E_m)} \langle\psi_0|m\rangle \langle n|\psi_0\rangle \langle m| Z |n\rangle \approx e^{-\beta(0+\Delta)} \langle\uparrow|0\rangle \langle 1|\uparrow\rangle \langle 0| Z |1\rangle$$

Techniques - Extracting (Global) Energy Gaps

$$\langle\uparrow| e^{-\beta H} Z e^{-\beta H} |\uparrow\rangle = \sum_{nm} e^{-\beta(E_n+E_m)} \langle\psi_0|m\rangle \langle n|\psi_0\rangle \langle m| Z |n\rangle \approx e^{-\beta(0+\Delta)} \langle\uparrow|0\rangle \langle 1|\uparrow\rangle \langle 0| Z |1\rangle$$



Techniques - Extracting (Global) Energy Gaps

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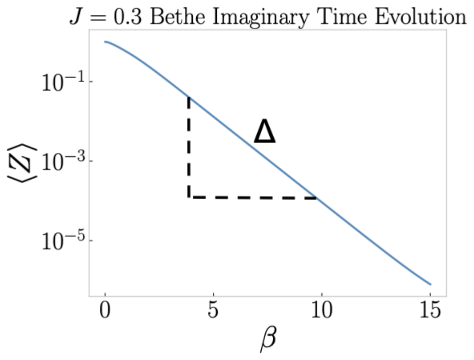
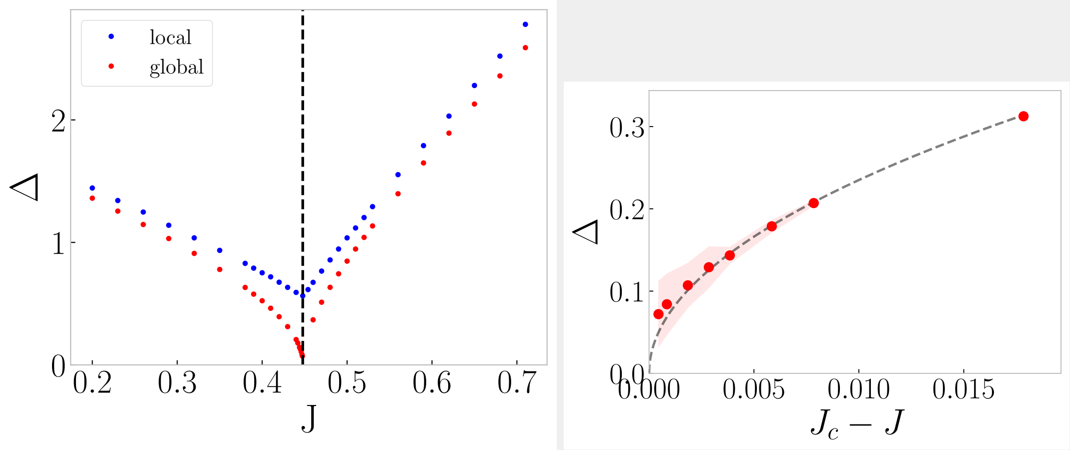


- Remark 1: Z used here as example in the disordered phase, but Y works everywhere.
- Remark 2: Cooling from globally distinct state gives Δ_{global}, kicking ground state locally and then cooling gives Δ_{local}.



- Local gap stays open, global gap closes ($\sim \sqrt{J_c - J}$) at transition.

Ongoing Work - Implications for Unitary State Preparation

- Time scales of adiabatic evolution are set by $1/\Delta$.
- $\Delta_{\text{local}} > 0$ suggests that long-range entangled states may be (unitarily) efficiently preparable on hyperbolic lattices!

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- Questions:
  - Can we sweep across the transition?

$$U(t) = \mathcal{T} \exp\left(i \int_0^t H(t')dt' \right)$$

  - Relative power of $t = O(1)$ local Hamiltonian evolution vs. $d = O(1)$ local circuits?

Conclusions/Takeaways

- (Local) Energy gaps don't have to close at phase transitions on hyperbolic lattices.
  - Consequences for unitary/adiabatic state preparation?
- Central tool: (MPS-like) Tree tensor networks.
- Main message: Hyperbolic space is a tractable setting to test our fundamental intuitions about quantum phases of matter.