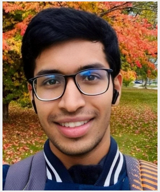
Efficient regimes of measurement-based quantum computation on a superconducting processor

EQUIPTNT Workshop, 7.10.2025

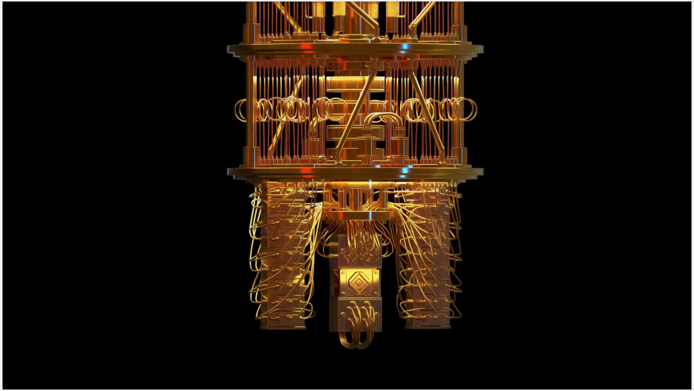
Rio Weil,
Arnab Adhikary, Dmytro Bondarenko, Robert Raussendorf



Featuring...

- *Measurement-based quantum computation in finite one-dimensional systems: string order implies computational power*, R. Raussendorf, W. Yang & A. Adhikary, Quantum 7, 1215 (2023)
- *Counterintuitive Yet Efficient Regimes for Measurement-Based Quantum Computation on Symmetry-Protected Spin Chains*, A. Adhikary, W. Yang, R. Raussendorf, Phys. Rev. Lett. 133, 160601 (2024)
- My MSc thesis + arXiv (soon...)

Motivating Questions

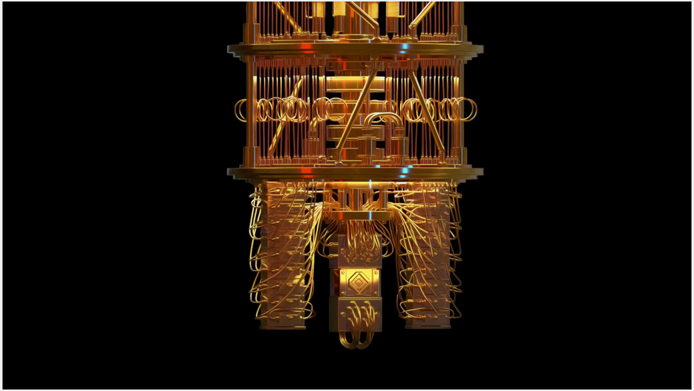


1. How do we characterize quantum computational speedup?

2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

Image Credit: Quanta Magazine

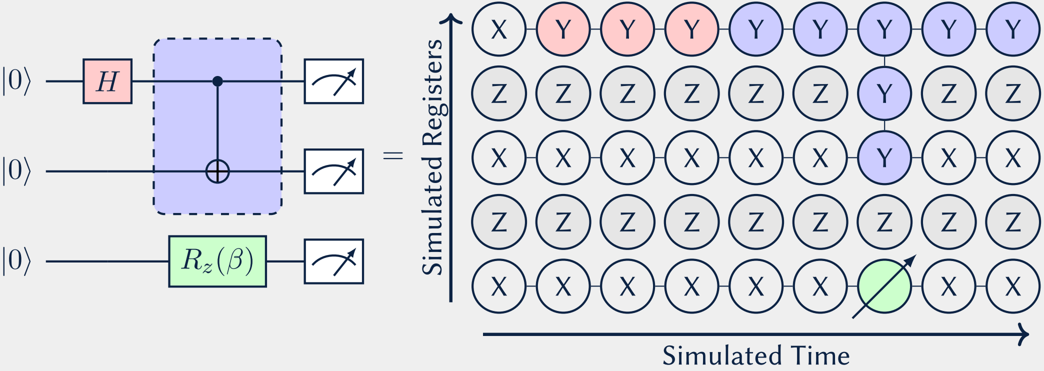
Motivating Questions



1. How do we characterize quantum computational speedup?
   - One route - Measurement-Based Quantum Computing
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Fun playground for physicists!

Image Credit: Quanta Magazine

A One-Slide Review of MBQC



	Circuit Model	MBQC
Initialization	$\|00\ldots0\rangle$	(Universal) resource state
Evolution	Unitary Gates	(Adaptive) single-qubit measurements

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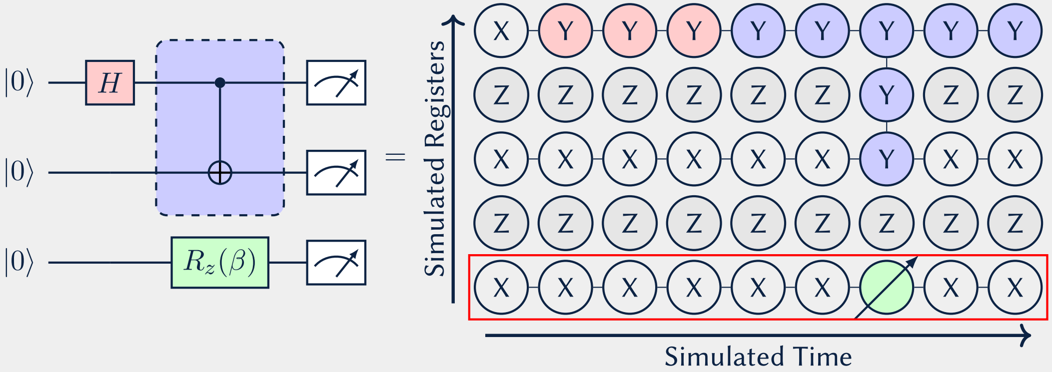

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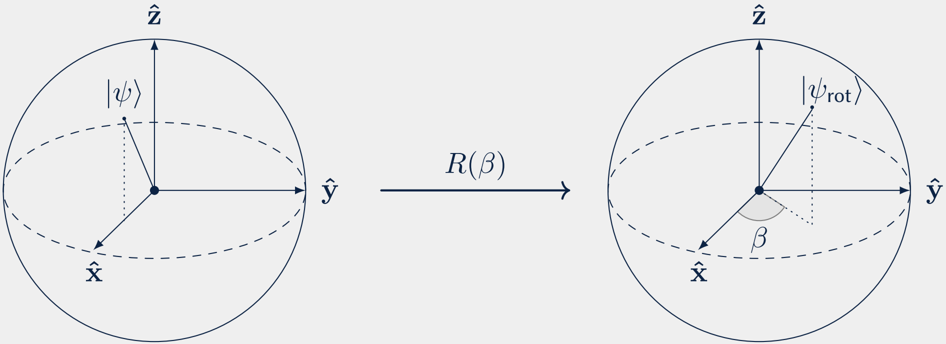
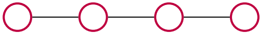


Ability to perform arbitrary single qubit unitaries (rotations) with high fidelity.

1D Resource States - 2 Examples and Interpolation

Universal Resource: Cluster State $|C\rangle$



Ground state of
$$H_{\text{cluster}} = -\sum_i Z_{i-1}X_iZ_{i+1}$$

Useless Resource: Product State $|+\rangle^{\otimes N}$

Ground state of
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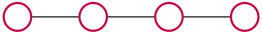

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Useless Resource: Product State $|+\rangle^{\otimes N}$

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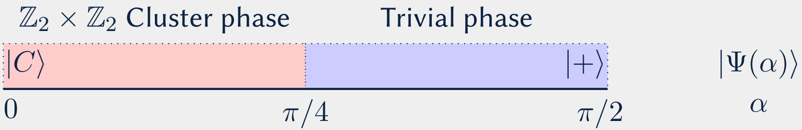
$H_{\text{product}} = -\sum_i X_i$

Computational order of ground states $|\Psi(\alpha)\rangle$ of:

$$H(\alpha) = -\cos(\alpha) \sum_i Z_{i-1} X_i Z_{i+1} - \sin(\alpha) \sum_i X_i$$

1D Resource States - SPT Phases & Decoherence

Answer (for infinite systems):



- Computational power is uniform in symmetry-protected topological (SPT) phases.

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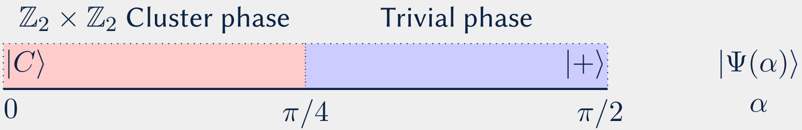

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$$|C\rangle \quad \overset{\text{symmetric FDLU}}{\not\longleftrightarrow} \quad |+\rangle$$

- Logical decoherence away from the cluster state!

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- Desired (logical) rotation $\exp\left(-i\frac{\beta}{2}P\right)$ becomes a probabilistic channel:

$$\mathcal{V} = \frac{1+\nu}{2}\left[\exp\left(-i\frac{\beta}{2}P\right)\right] + \frac{1-\nu}{2}\left[\exp\left(i\frac{\beta}{2}P\right)\right]$$

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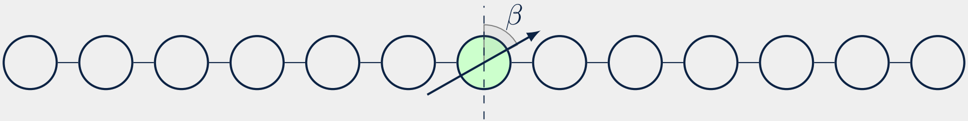
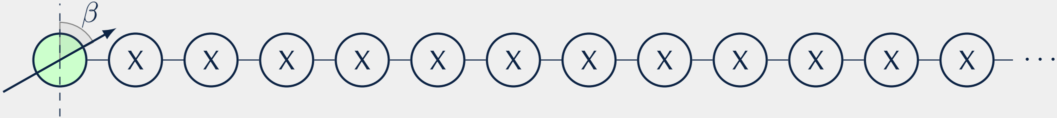
- Equivalent to σ the *string order parameter.*

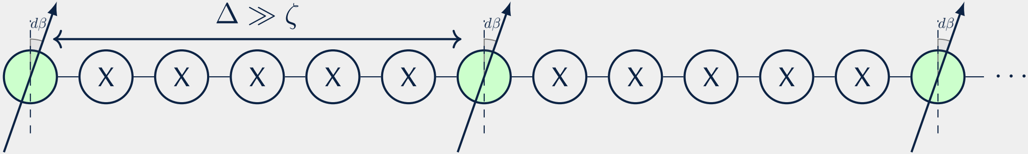


$$\sigma = \langle Z \quad X \quad I \quad X \quad I \quad X \quad Z \rangle$$

1D Resource States - Decoherence Management I

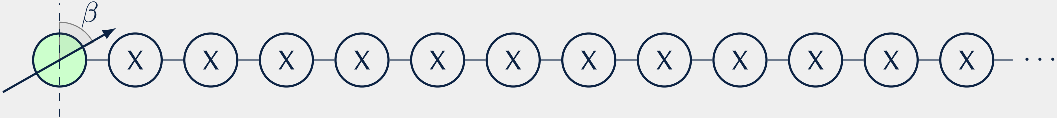


vs.



- Error is $O(\beta^2)$ - m subdivisions reduces error:

$$\epsilon_{\text{tot}} = m \cdot \epsilon_{\text{single}} \sim m \cdot \left(\frac{\beta}{m}\right)^2 = \frac{\beta^2}{m}$$



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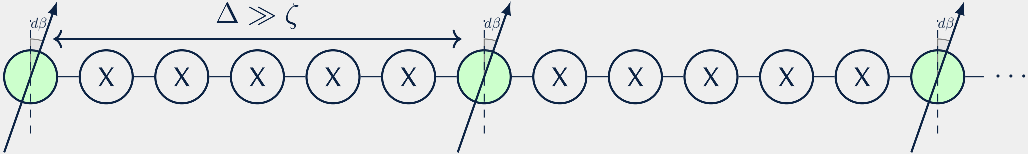

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- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as needed $\Rightarrow$ computational phases.

- Finite case: Tradeoff of rotation splitting and independence.

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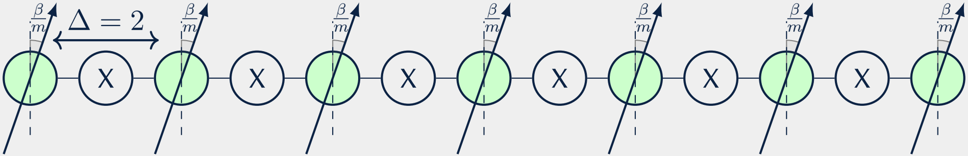

- Optimal strategy: Splitting wins!

Proposed Experiments

1. Computational order = String order
2. Decoherence management I - Divide and conquer
3. Decoherence management II - The counterintuitive regime

Experiment 0 - Ground State Anstatz

Recall $H(\alpha)$:

$$H(\alpha) = -\cos(\alpha) \sum_i Z_{i-1} X_i Z_{i+1} - \sin(\alpha) \sum_i X_i$$

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We consider the following variational ansatz:

$$|\psi(\theta)\rangle = \bigotimes_{i=2}^{N-1} T_i(\theta)\,|\mathcal{C}\rangle = \bigotimes_{i=2}^{N-1}(\cos(\theta)I_i + \sin(\theta)X_i)\,|\mathcal{C}\rangle$$

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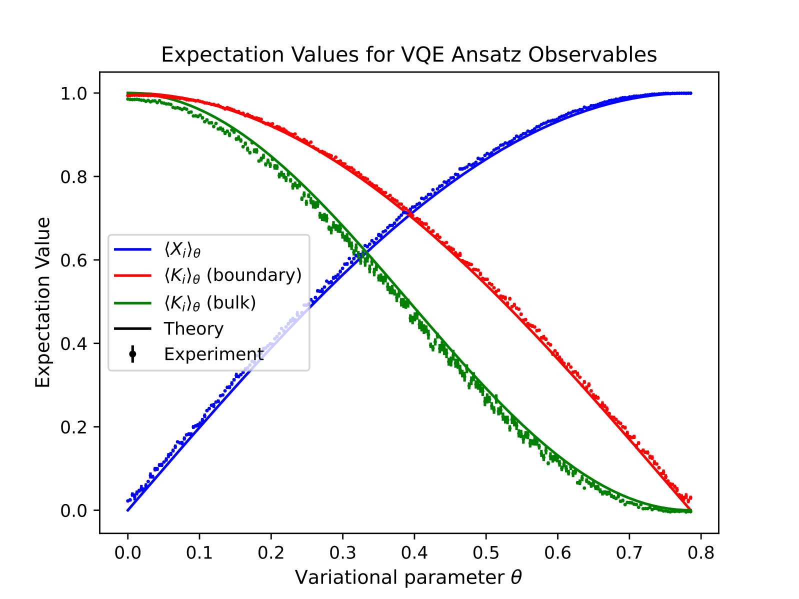
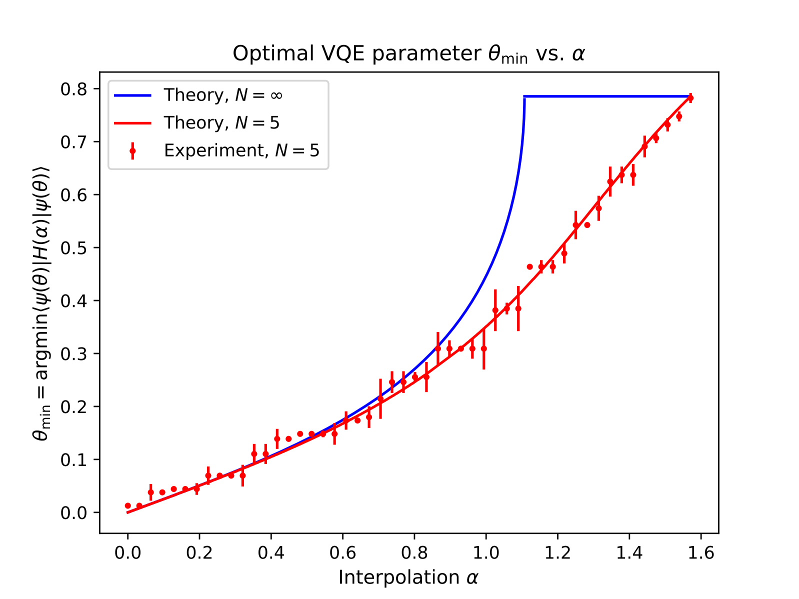
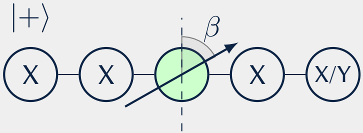
For a given value of α, find $|\psi(\theta)\rangle$ which minimizes:

$$\langle\psi(\theta)|\,H(\alpha)\,|\psi(\theta)\rangle = -\cos(\alpha)\sum_{i=1}^{N}\langle K_i = Z_{i-1}X_iZ_{i+1}\rangle_\theta - \sin(\alpha)\sum_{i=1}^{N}\langle X_i\rangle_\theta$$

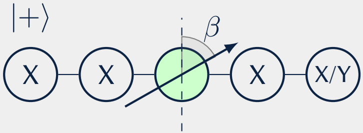
Experiment 0 - State Preparation (Results)




Optimal VQE parameter $\theta_{\min}$ vs. α
Theory, $N = \infty$
Theory, $N = 5$
Experiment, $N = 5$
$\theta_{\min} = \operatorname{argmin}\langle\psi(\theta)|H(\alpha)|\psi(\theta)\rangle$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Interpolation α



$$\sigma = \langle Z_3 \quad X_4 \quad Z_5 \rangle$$



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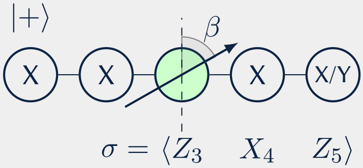
(from $\mathcal{V}$): $\langle \overline{X} \rangle_+ = \cos(\beta), \langle \overline{Y} \rangle_+ = \nu \sin(\beta) \implies \dfrac{\langle \overline{Y} \rangle_+}{\langle \overline{X} \rangle_+} = \nu \tan(\beta)$



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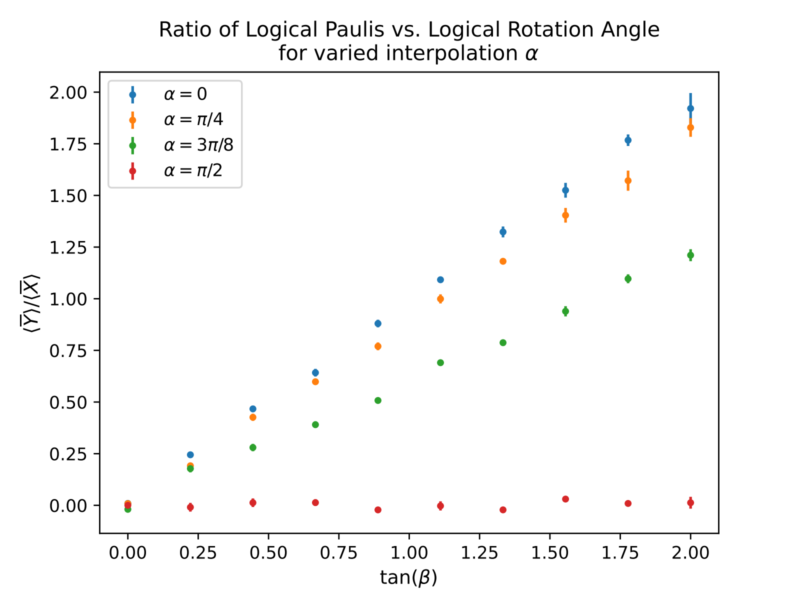
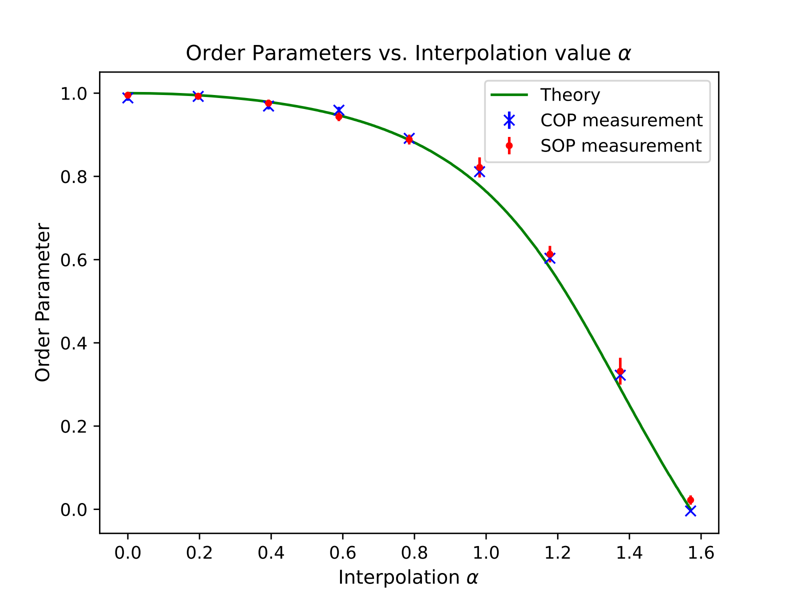
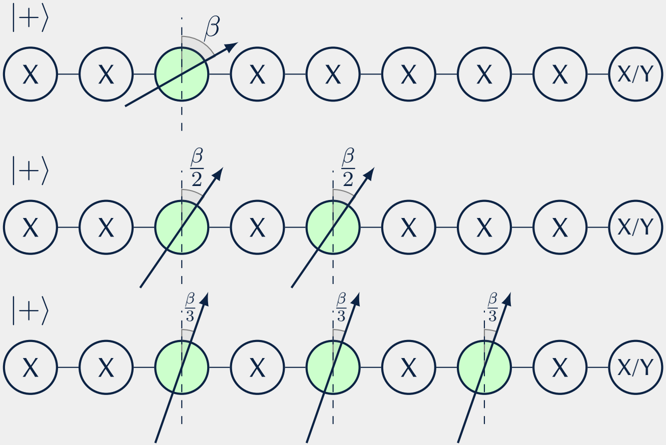
ν from MBQC, σ (for free) from VQE!

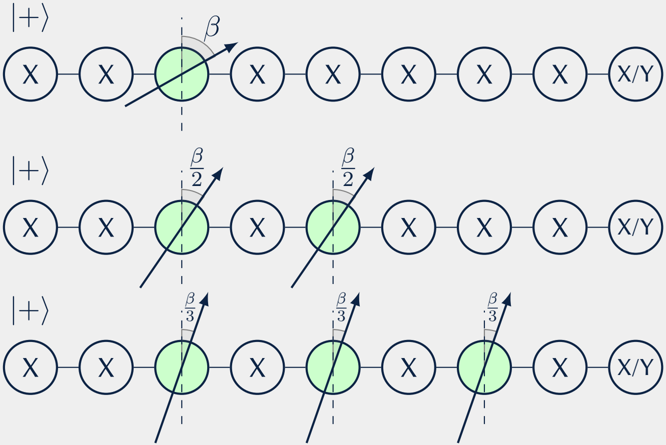


Ratio of Logical Paulis vs. Logical Rotation Angle
for varied interpolation α
α = 0
α = π/4
α = 3π/8
α = π/2
⟨Ȳ⟩/⟨X̄⟩
tan(β)
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

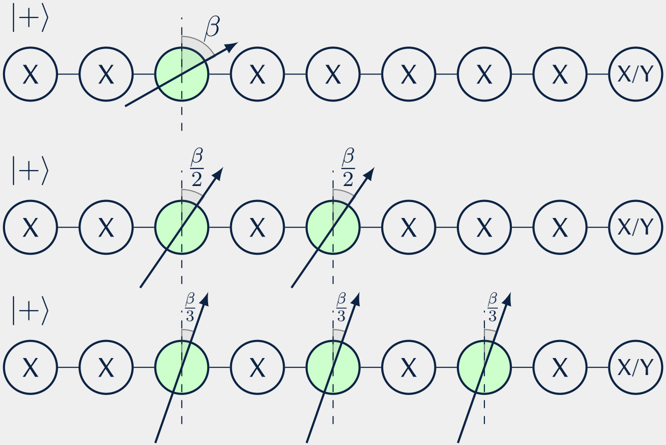
Experiment 1 - Computational order = String order (Results)




|+⟩
β
X
X
X
X
X
X
X
X/Y
|+⟩
β/2
β/2
X
X
X
X
X
X/Y
|+⟩
β/3
β/3
β/3
X
X
X
X
X/Y



- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle \overline{X}(\beta) \rangle^2 - \langle \overline{Y}(\beta) \rangle^2$ in the three cases.



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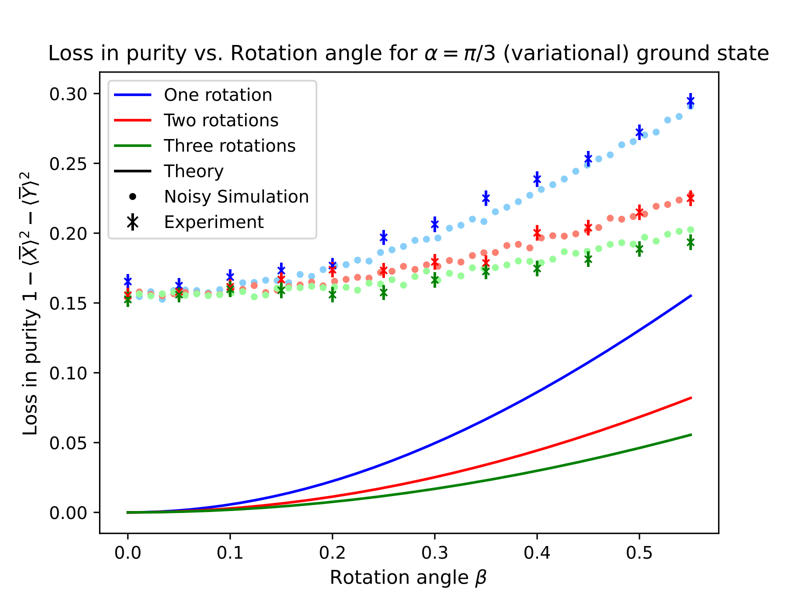
- For small angles β, verify $\text{LOP} \sim \frac{1}{m}$ (from $\mathcal{V}$).

Experiment 2 - Divide and conquer (Results)



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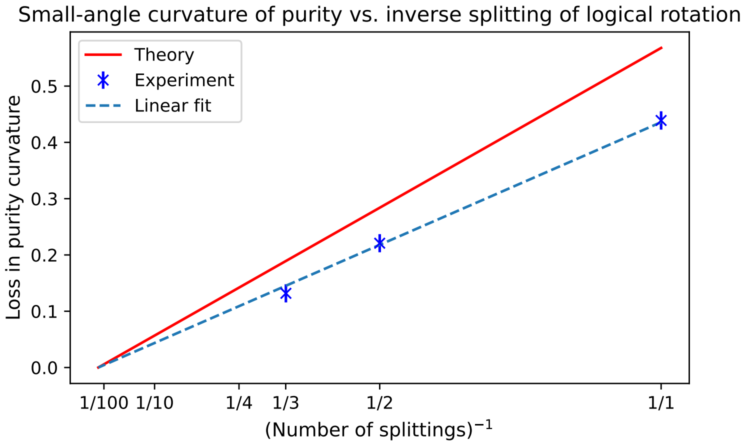

Experiment 3 - Resource state

- VQE ansatz $|\psi(\theta)\rangle$ has no length scale!

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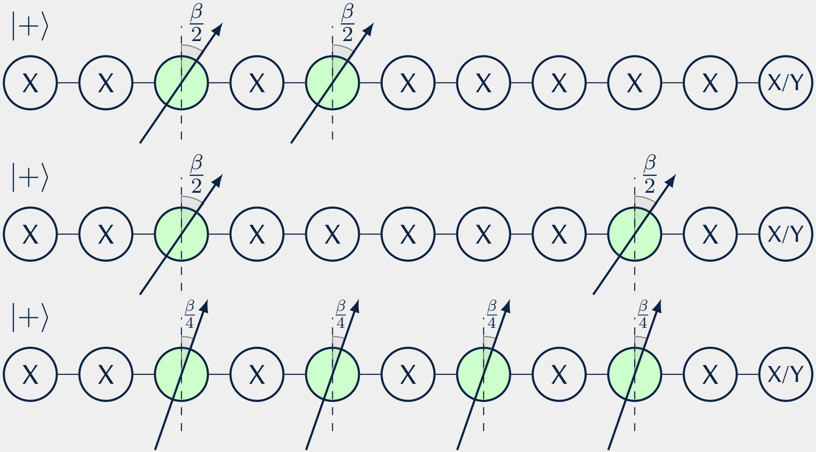
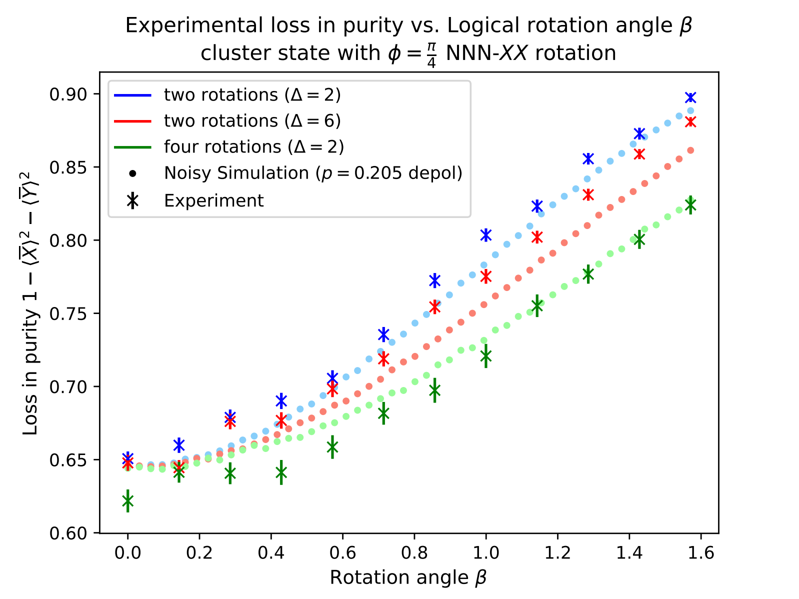
- Instead, we consider:

$$|\Omega(\phi)\rangle = \prod_{i=2}^{N-3} RXX_{i,i+2}(\phi) \, |\mathcal{C}\rangle$$



- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle \overline{X}(\beta) \rangle^2 - \langle \overline{Y}(\beta) \rangle^2$ in the three cases.

Experiment 3 - The counterintutive regime (Results)



Conclusions

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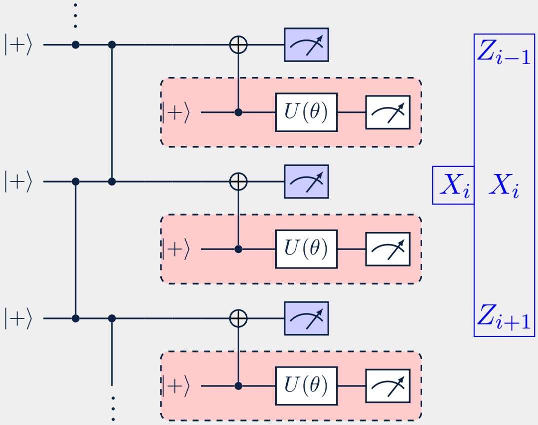
Outlook: How do we get more out of these devices?

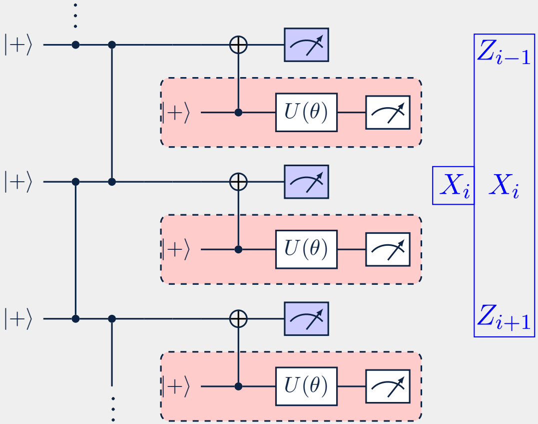
Experiment 0 - VQE for Ground state



Algorithm for finding $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$:

1. Prepare the cluster state $|\mathcal{C}_N\rangle$.
2. Probabilistically implement (non-unitary) $T_i(\theta) = \cos(\theta) I_i + \sin(\theta) X_i$ on each site.
3. Measure X_i or $K_i = Z_{i-1} X_i Z_{i+1}$ on the prepared state to obtain $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$.

Experiment 0 - VQE for Ground state



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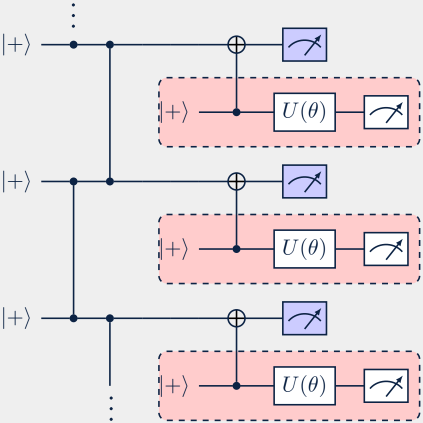
Then, various tricks with symmetry, half-teleportation, translation invariance...

Experiment 0 - VQE Simplifications



$$O(n)$$

Experiment 0 - VQE Simplifications