

QUANTUM PHASES OF MATTER IN HYPERBOLIC SPACE

UCHICAGO-UTOKYO QIT WORKSHOP

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MOTIVATION - WHAT IS HYPERBOLIC SPACE?

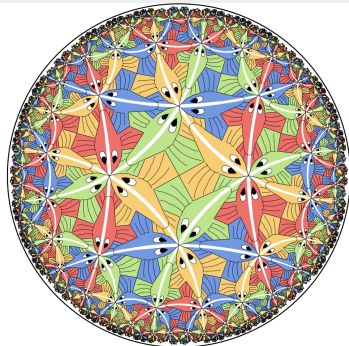
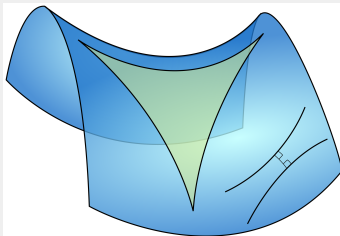


Image Credits: Reefguide; Wikipedia; D. Dunham (Transformation of Hyperbolic Escher Patterns)

MOTIVATION - GENERAL SURVEY

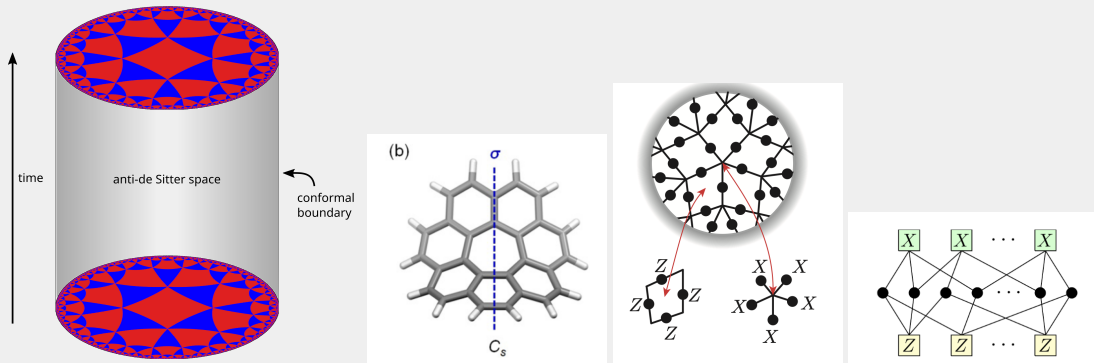
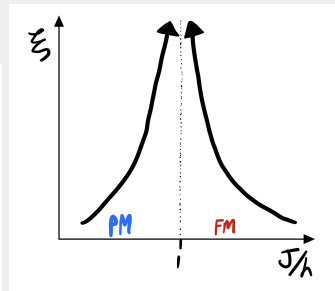
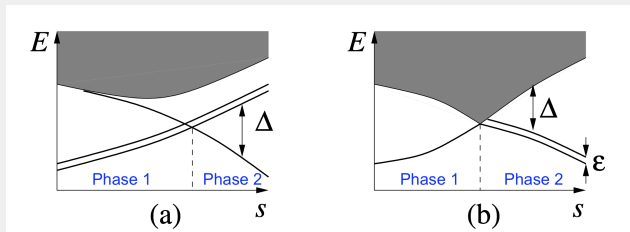


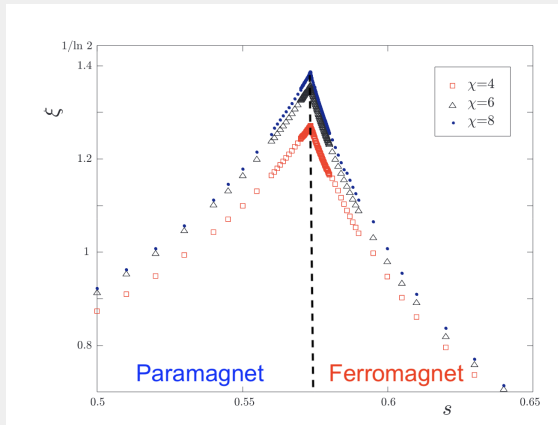
Image Credits: Wikipedia; Org. Lett. 2017, 19, 9, 2246-2249; arXiv:1703.00590; Quantum 5, 585 (2021)

MOTIVATION - SPECIFIC QUESTIONS



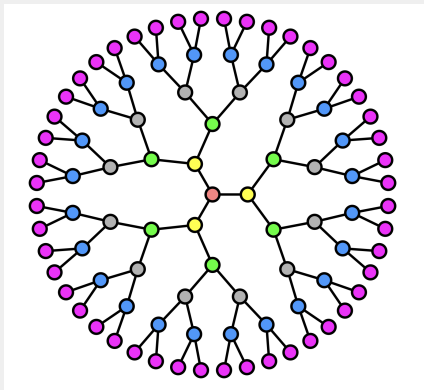
- Euclidean: Established relationships between adiabaticity/phases/gaps/correlations.

MOTIVATION - SPECIFIC QUESTIONS



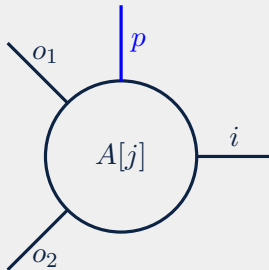
- Correlations and Energetics: Euclidean intuitions can be broken...
 - ▶ No Goldstone bosons [Lauman et. al, *Phys. Rev. B* (2009)]
 - ▶ Non-divergent correlation length at phase transition [Nagaj et. al, *Phys. Rev. B* (2008)]
- Efficient preparability of states?

SETTING - TFIM ON CAYLEY TREE/BETHE LATTICE

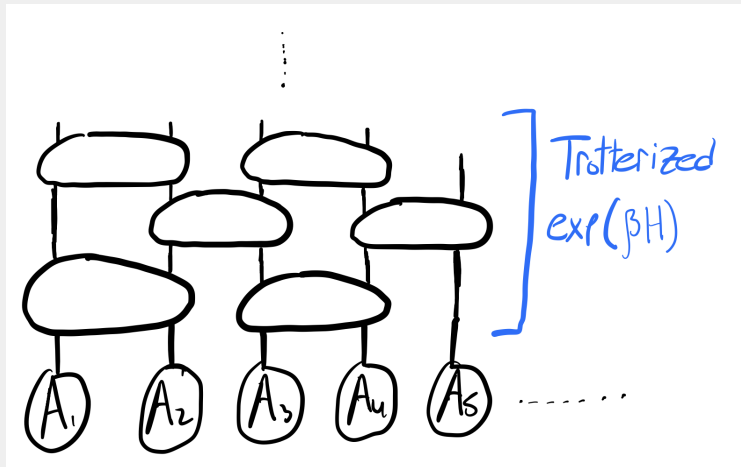


$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_{i \text{ bulk}} X_i - g_{\text{bdy}} \sum_{i \text{ boundary}} X_i$$

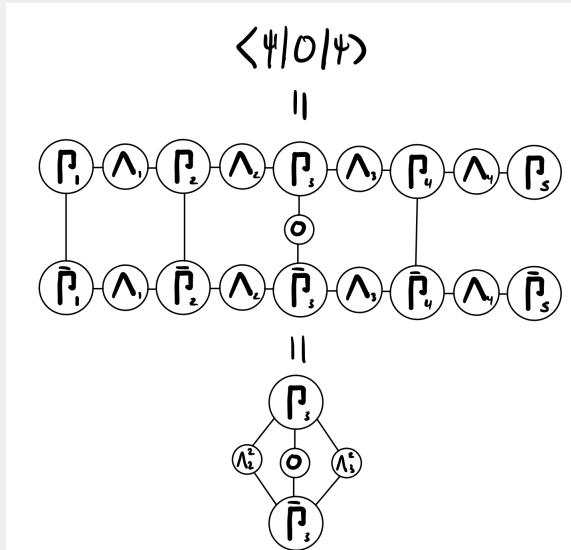
Image Credit: arXiv:1406.2819



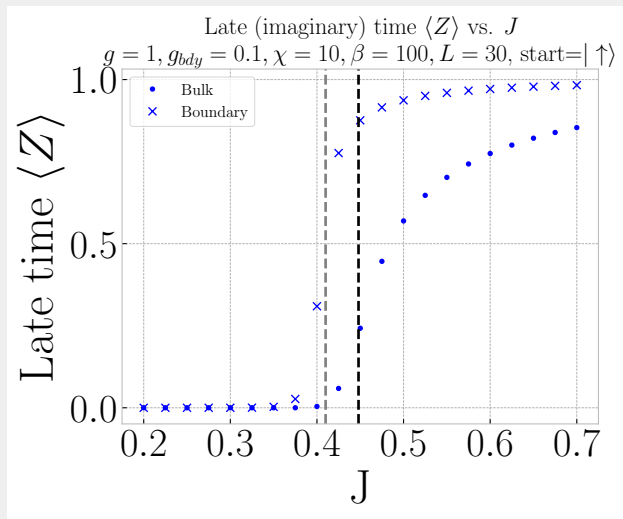
- Rotational symmetry $\implies O(L)$ tensors for L rings ($\sim 2^L$ (!) qubits!)



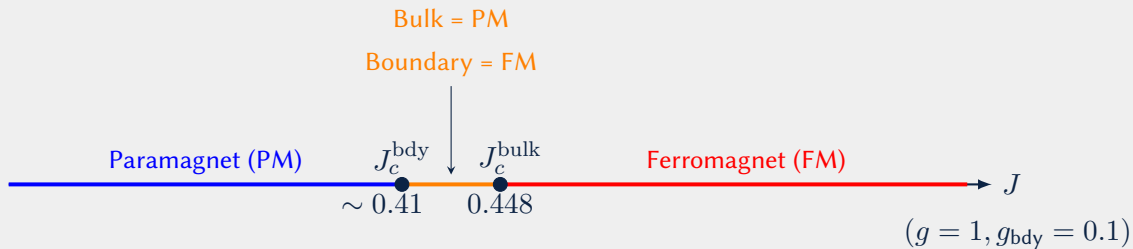
- Trotterized $\exp(\beta H)$ or $\exp(itH)$ can be applied to simulate time evolution



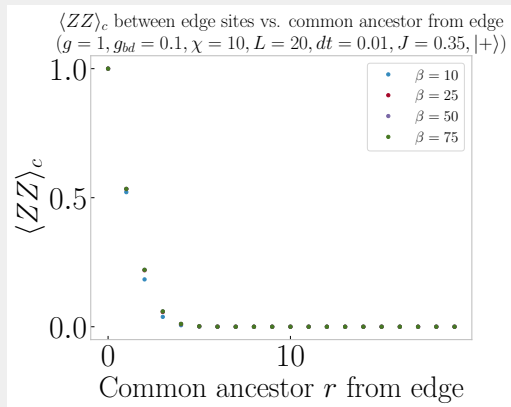
- Canonical form makes computing local expectation values efficient/stable.



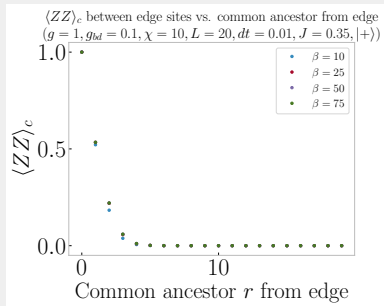
RESULTS - PHASE DIAGRAM



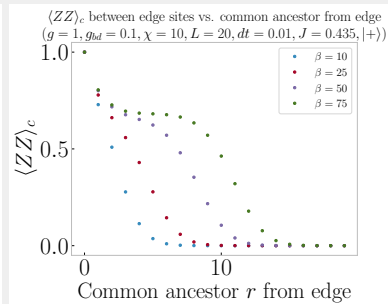
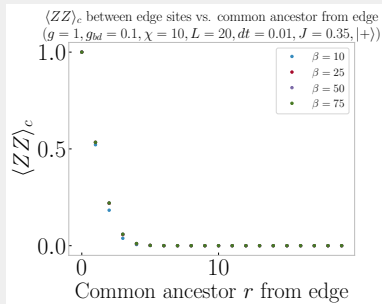
RESULTS - STATIC SPATIAL CORRELATIONS (CAT STATE GROWTH)



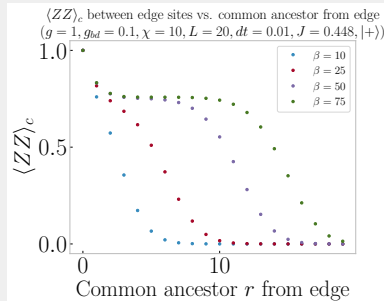
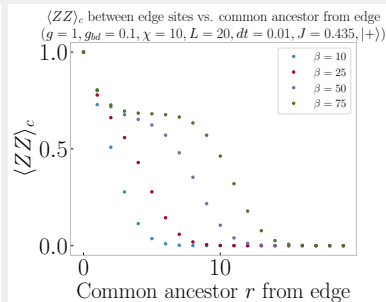
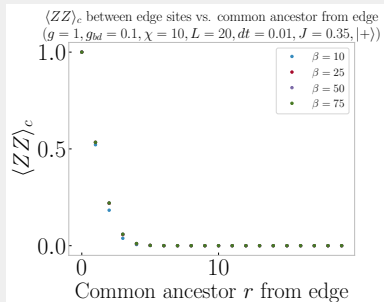
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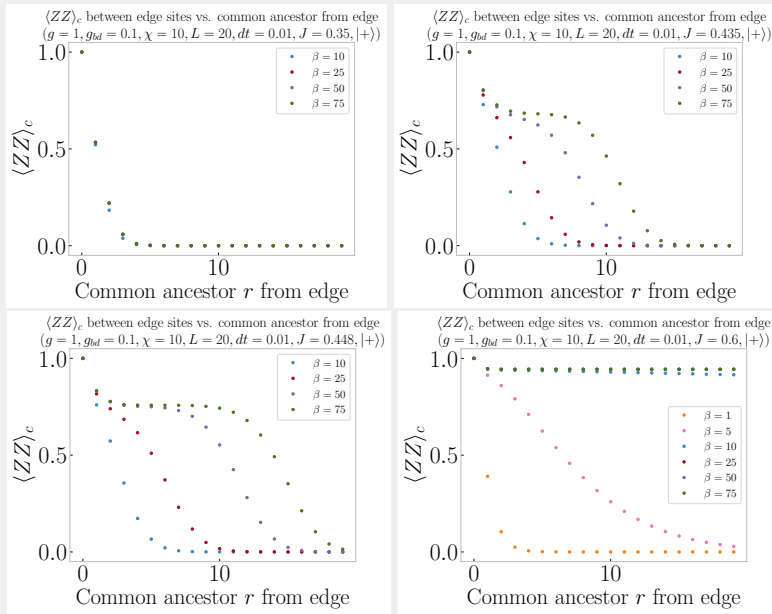
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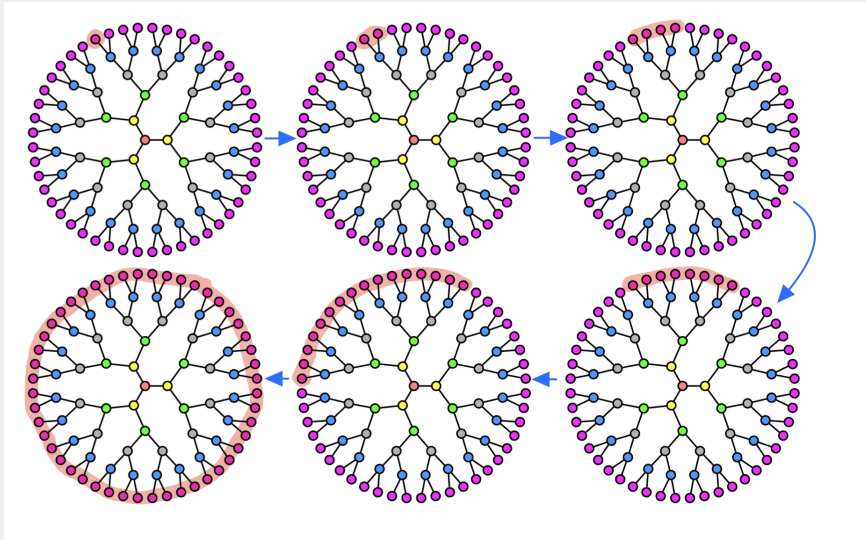
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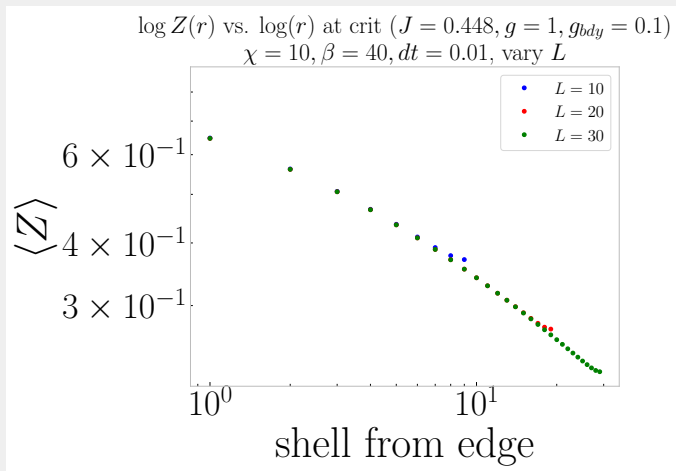
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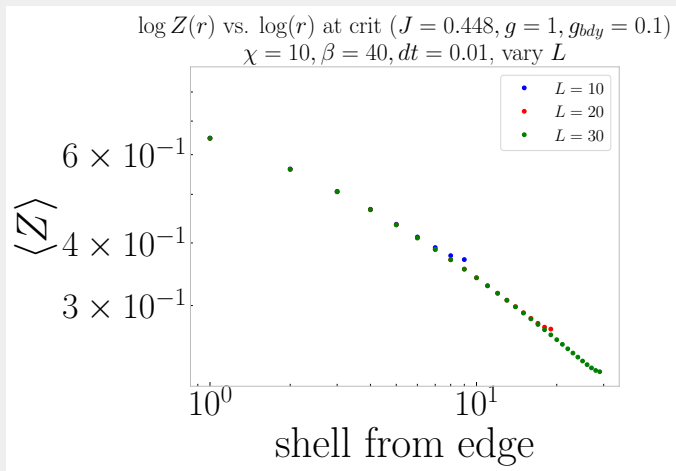


RESULTS - STATIC SPATIAL CORRELATIONS (ALGEBRAIC DECAY)



- Tells us about $\langle Z_0 Z(r) \rangle_c = \langle Z_0 Z(r) \rangle - \langle Z_0 \rangle \langle Z(r) \rangle = \langle Z_0 Z(r) \rangle = C \langle Z(r) \rangle$.

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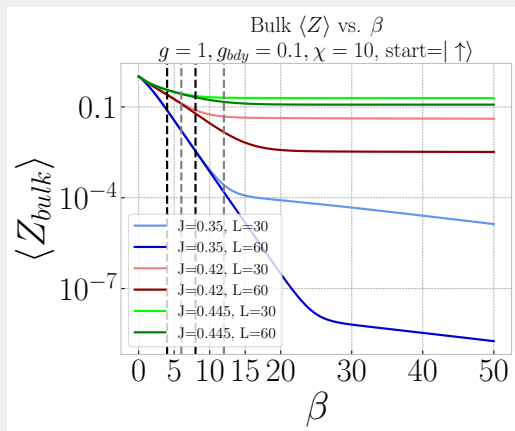
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- Algebraic, not exponential (as in Bethe case)!

RESULTS - SPECTRUM FROM DYNAMIC CORRELATIONS (Z)

$$\langle \uparrow | e^{-\beta H} Z e^{-\beta H} | \uparrow \rangle = \sum_{nm} e^{-\beta(E_n + E_m)} \langle \uparrow | m \rangle \langle n | \uparrow \rangle \langle m | Z | n \rangle \approx e^{-\beta(0 + \Delta)} \langle \uparrow | 0 \rangle \langle 1 | \uparrow \rangle \langle 0 | Z | 1 \rangle$$

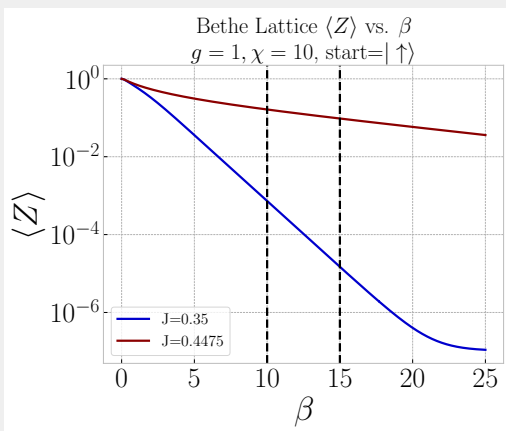
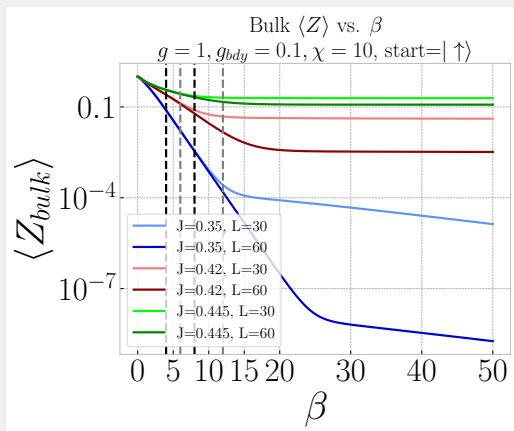
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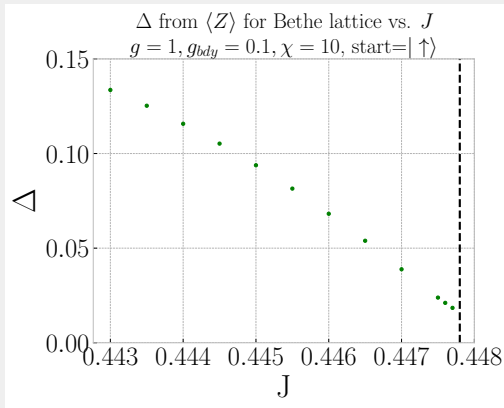
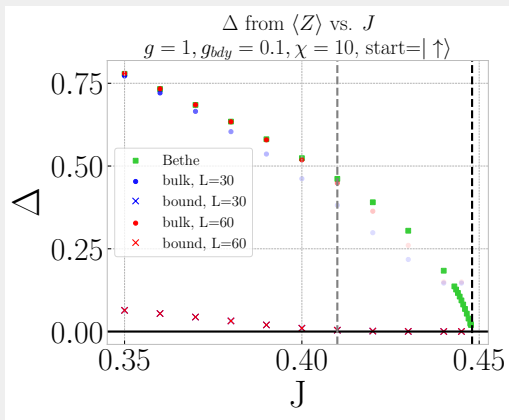


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RESULTS - SPECTRUM FROM DYNAMIC CORRELATIONS (Z)



- Gapless at boundary transition, Gapped at boundary transition(?)

- Hastings-type bounds [*arXiv/1008.5137*]: Gap $\Delta \implies \langle AB \rangle_c \leq O(e^{-\Delta})$
- Technical subtlety; bound on:

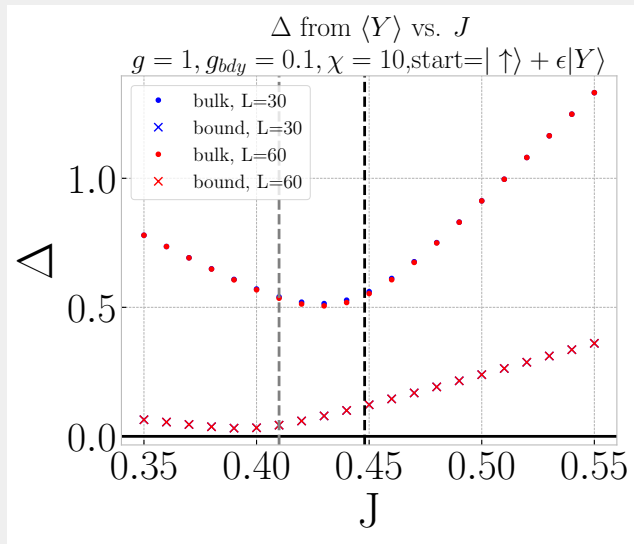
$$\langle AB \rangle_c = \langle \psi_0 | AB | \psi_0 \rangle - \langle \psi_0 | AP_0 B | \psi_0 \rangle$$

with:

$$P_0 = \sum_a |\psi_0^a\rangle\langle\psi_0^a|$$

- With P_0 terms, the decay looks exponential (theorem is safe!) but the “physical” correlator decays algebraically.

RESULTS - SPECTRUM FROM DYNAMIC CORRELATIONS (Y)



MOTIVATION - MEASUREMENT-BASED STATE PREP

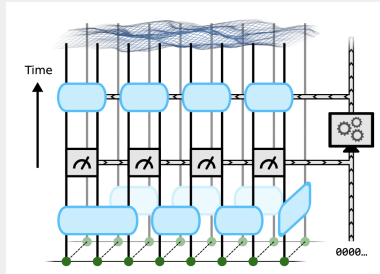
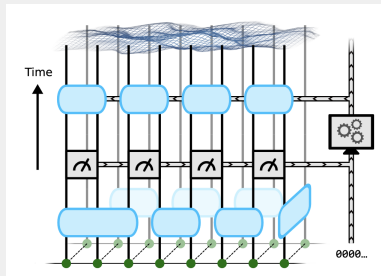


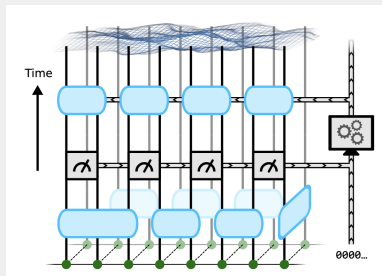
Image credit: PRX Quantum 3, 040337

MOTIVATION - MEASUREMENT-BASED STATE PREP



- LRE states in constant time (GHZ [Briegel & Raussendorf, *PRL* (2001)], Toric code [Raussendorf, Brayvi & Hastings, *PRA* (2005)],...)

MOTIVATION - MEASUREMENT-BASED STATE PREP



- LRE states in constant time (GHZ [Briegel & Raussendorf, PRL (2001)], Toric code [Raussendorf, Brayvi & Hastings, PRA (2005)],...)
- Extended to non-stabilizer states, variable correlation length states, e.g.:

$$|\Psi_\beta\rangle = \exp(\beta H_{\text{TC}}) |0\rangle^{\otimes N} \sim \exp\left(\beta \prod_s X_s\right) |0\rangle^{\otimes N}$$

(many works... [arXiv/2404.16083, 2404.16360, 2404.16753, 2404.17087, 2405.09615])

- What about critical states? E.g. (via map to Ising):

$$|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$$

- The landscape:

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 - ▶ 1-D: No transition, but easily preparable [*Sahay and Veressen, PRX Quantum (2025)*]

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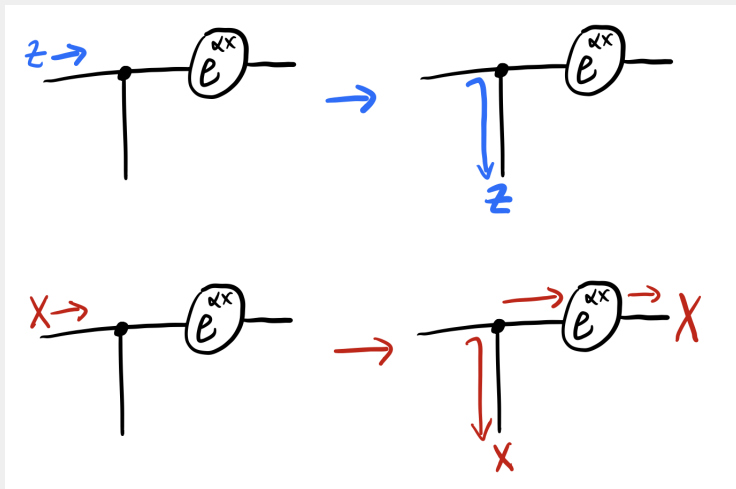
- The landscape:
 - ▶ 1-D: No transition, but easily preparable [*Sahay and Veressen, PRX Quantum (2025)*]
 - ▶ 2-D Square: Transition, but not easily preparable [*Zhu et al, PRL (2023)*]
 - ▶ Trees: Transition to “boundary sensitive” phase [*Eggarter, PRB (1974), Wang et al. PRB (2025)*] + preparable!

TECHNIQUES - PREPARING $\exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$ (STEP 1)

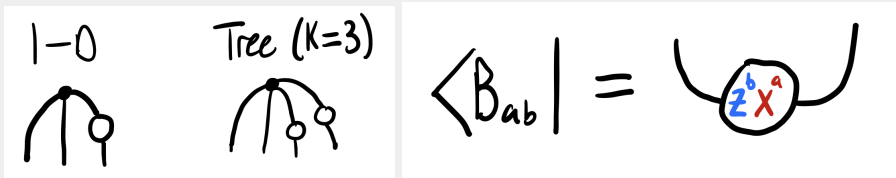
The diagram shows an equality between two interaction terms. On the left, two vertical lines represent qubits, with an oval between them containing the expression $e^{\beta z_i z_j}$. On the right, two horizontal lines represent qubits, with an oval between them containing the expression $e^{\alpha x_i x_j}$. The two diagrams are separated by an equals sign.

- With $\tanh(\beta) = e^{-2\alpha}$; both are $\text{diag}(e^{\beta}, e^{-\beta}, e^{-\beta}, e^{\beta})$.

TECHNIQUES - PREPARING $\exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$ (STEP 2)



TECHNIQUES - PREPARING $\exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$ (STEP 3)

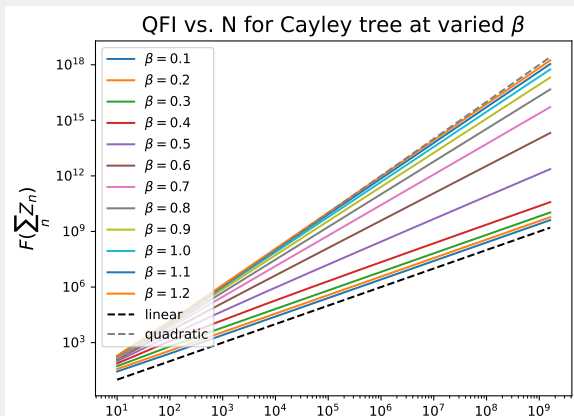


- 3/4-qubit spiders connected via Bell measurements
- Push errors to boundary, then single layer of cleanup!

OBSERVING THE (UNCONVENTIONAL) PHASE TRANSITION

- Predicted transition at $\beta_c = \frac{1}{2} \ln\left(\frac{q}{q-2}\right) \approx 0.55$ can be observed from the QFI of the state:

$$\text{QFI}_\beta\left(\sum_i Z_i\right) = \left\langle \sum_{ij} Z_i Z_j \right\rangle_\beta - \left\langle \sum_i Z_i \right\rangle_\beta^2$$



- (Simple) spin systems in hyperbolic space can host unintuitive fundamental phenomena
 - ▶ Boundary sensitive phase diagrams
 - ▶ New interplays of gaps and correlations!
- Protocols for (elusive) efficient construction of critical states!