Quantum Phases of Matter in Hyperbolic Space

UChicago-UTokyo QIT Workshop

Rio Weil, Ruben Verresen

Department of Physics, The University of Chicago
Priziker School of Molecular Engineering, The University of Chicago

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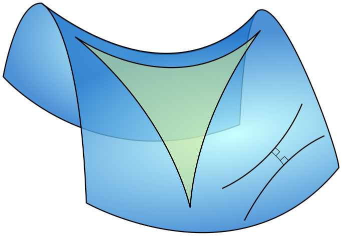
Motivation - What is hyperbolic space?



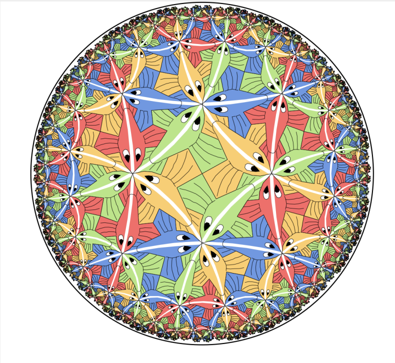
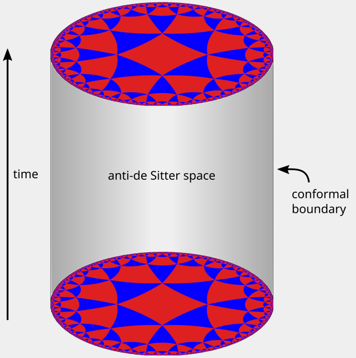
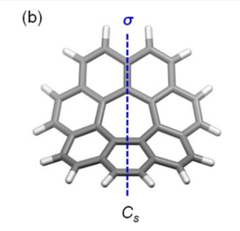
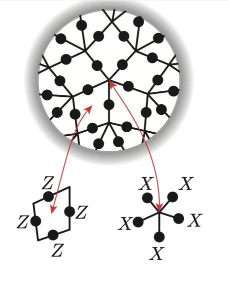


Image Credits: Reefguide; Wikipedia; D. Dunham (Transformation of Hyperbolic Escher Patterns)







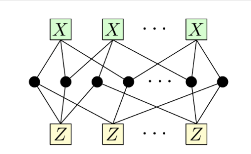
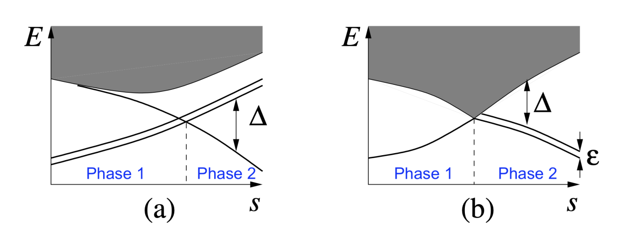
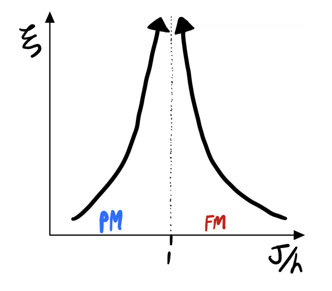


Image Credits: Wikipedia; Org. Lett. 2017, 19, 9, 2246-2249; arXiv:1703.00590; Quantum 5, 585 (2021)

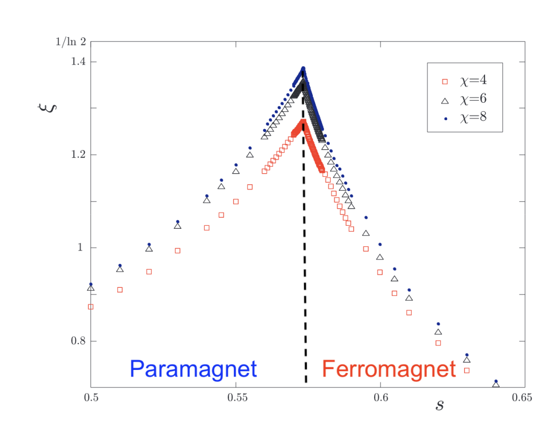
Motivation - Specific Questions





- Euclidean: Established relationships between adiabaticity/phases/gaps/correlations.

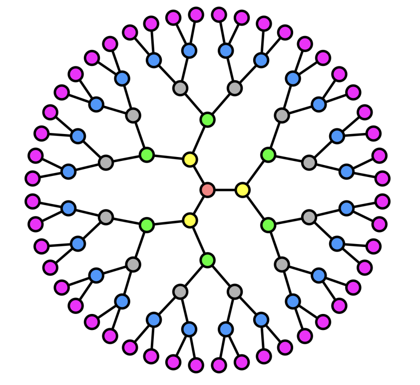
Image Credit: Phys. Rev. B 82, 155138



- Correlations and Energetics: Euclidean intuitions can be broken...
  - No Goldstone bosons *[Lauman et. al, Phys. Rev. B (2009)]*
  - Non-divergent correlation length at phase transition *[Nagaj et. al, Phys. Rev. B (2008)]*
- Efficient preparability of states?

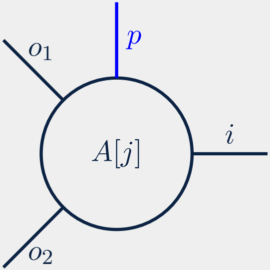
Image Credit: Phys. Rev. B 77, 214431

Setting - TFIM on Cayley Tree/Bethe Lattice

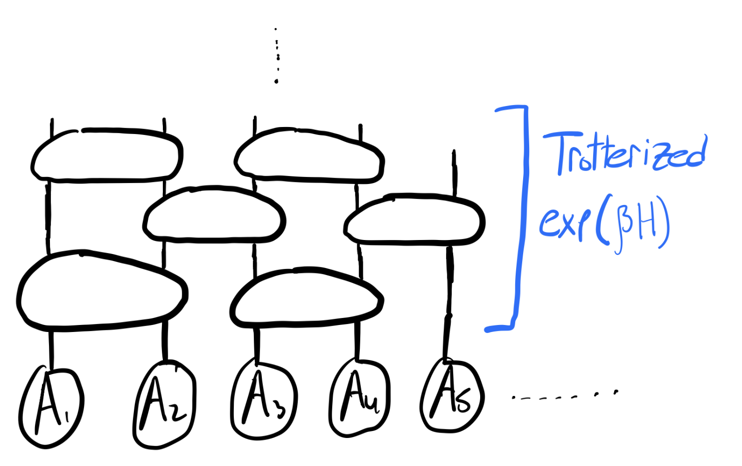


$$H = -J\sum_{\langle ij\rangle} Z_i Z_j - g\sum_{i \text{ bulk}} X_i - g_{\text{bdy}} \sum_{i \text{ boundary}} X_i$$

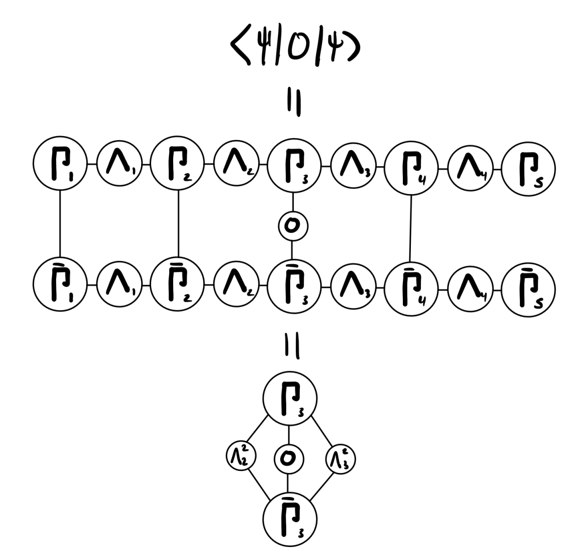
Image Credit: arXiv:1406.2819



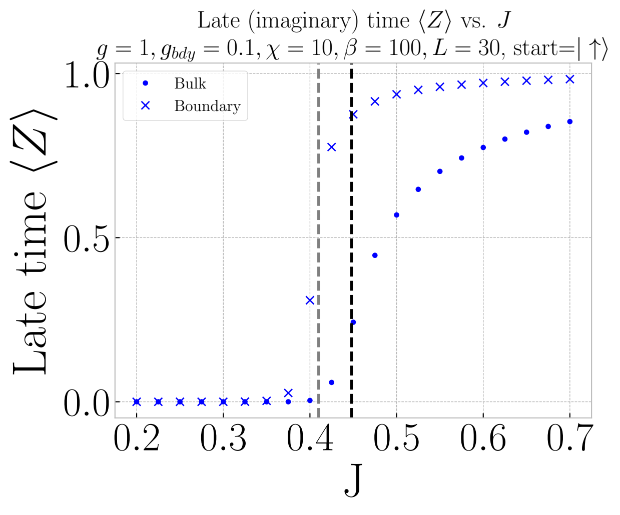
- Rotational symmetry $\implies$ $O(L)$ tensors for L rings ($\sim 2^L$(!) qubits!)



- Trotterized $\exp(\beta H)$ or $\exp(itH)$ can be applied to simulate time evolution

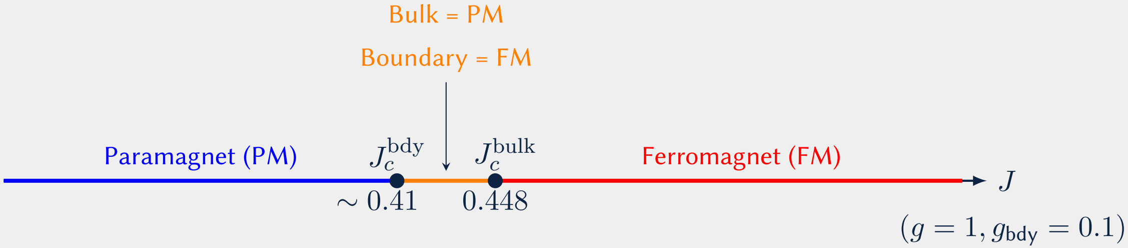


- Canonical form makes computing local expectation values efficient/stable.

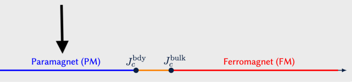


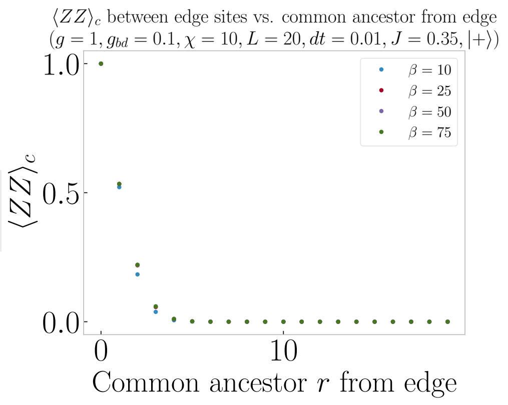
Late (imaginary) time ⟨Z⟩ vs. J
g = 1, g_bdy = 0.1, χ = 10, β = 100, L = 30, start=| ↑⟩
Bulk
Boundary
Late time ⟨Z⟩
1.0
0.5
0.0
0.2
0.3
0.4
0.5
0.6
0.7
J

Results - Phase Diagram



Results - Static Spatial Correlations (Cat State Growth)





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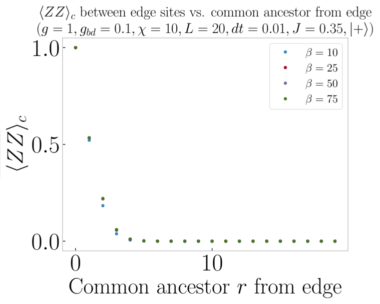

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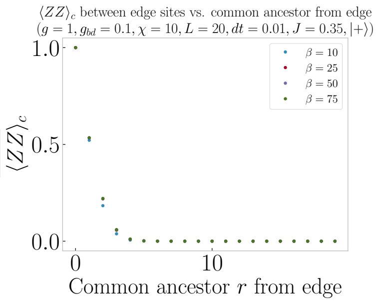
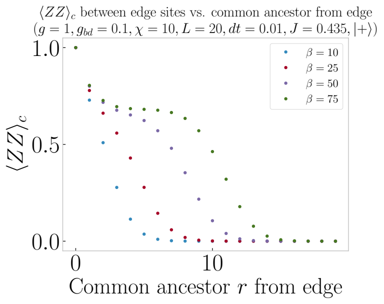
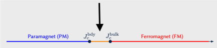
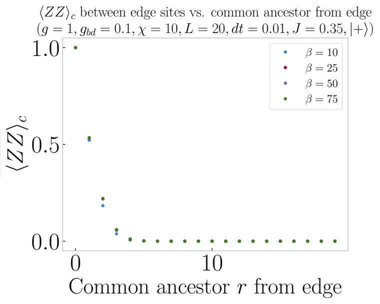
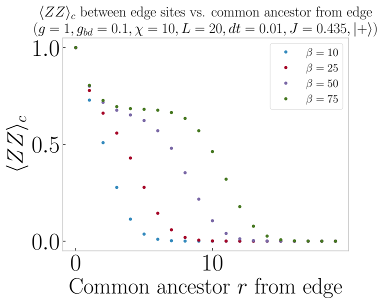


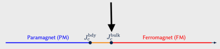


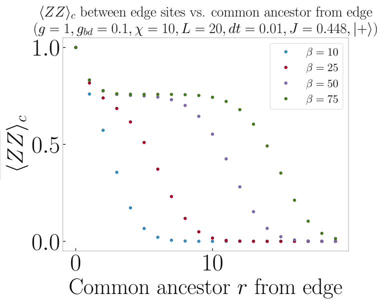


RESULTS - STATIC SPATIAL CORRELATIONS (CAT STATE GROWTH)









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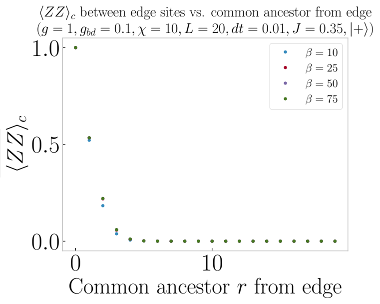

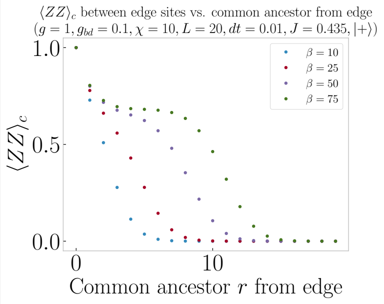

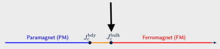

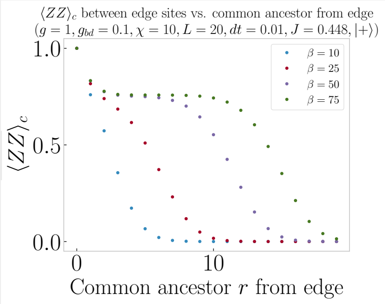

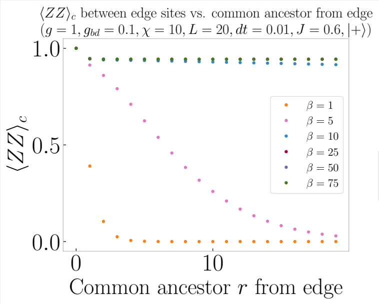

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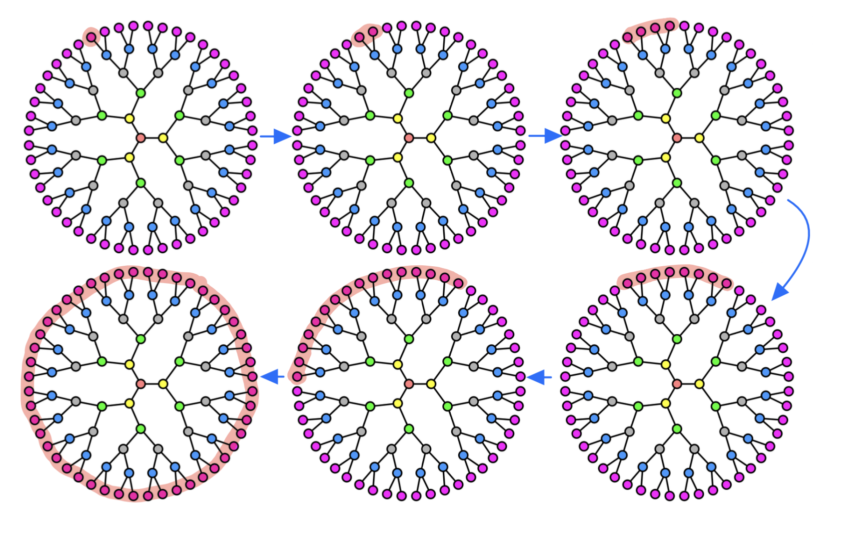
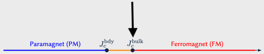
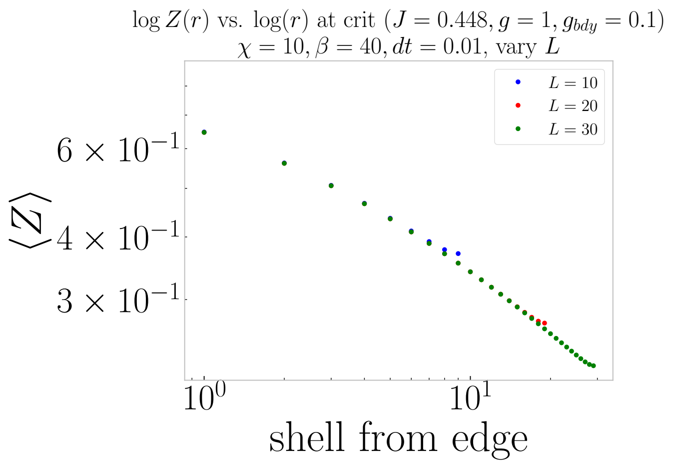


Results - Static Spatial Correlations (Algebraic Decay)





- Tells us about $\langle Z_0 Z(r) \rangle_c = \langle Z_0 Z(r) \rangle - \langle Z_0 \rangle \langle Z(r) \rangle = \langle Z_0 Z(r) \rangle = C \langle Z(r) \rangle$.

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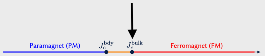

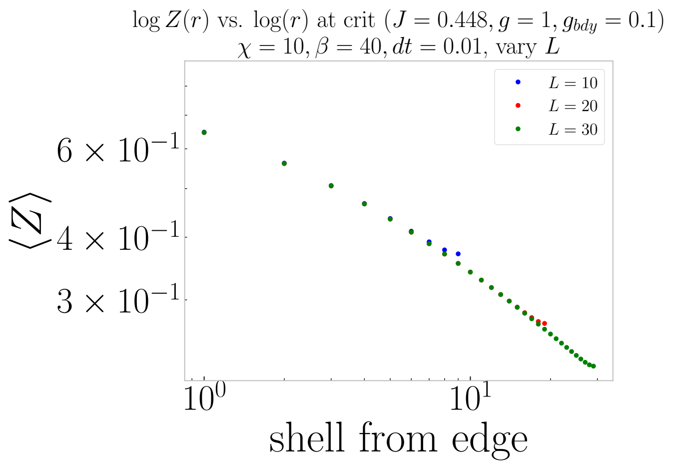

- Tells us about $\langle Z_0 Z(r)\rangle_c = \langle Z_0 Z(r)\rangle - \langle Z_0\rangle\langle Z(r)\rangle = \langle Z_0 Z(r)\rangle = C\langle Z(r)\rangle$.
- Algebraic, not exponential (as in Bethe case)!

Results - Spectrum from Dynamic Correlations (Z)

$$\langle\uparrow| e^{-\beta H} Z e^{-\beta H} |\uparrow\rangle = \sum_{nm} e^{-\beta(E_n+E_m)} \langle\uparrow|m\rangle \langle n|\uparrow\rangle \langle m| Z |n\rangle \approx e^{-\beta(0+\Delta)} \langle\uparrow|0\rangle \langle 1|\uparrow\rangle \langle 0| Z |1\rangle$$

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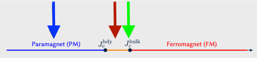

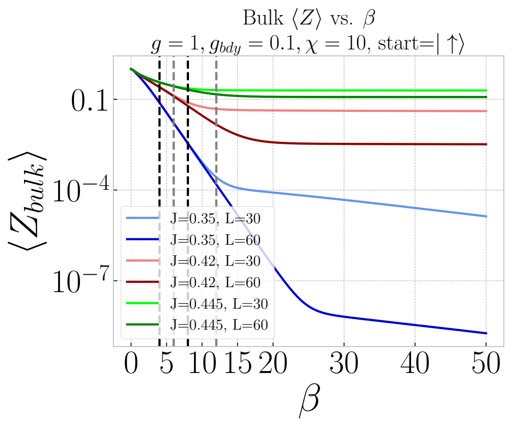

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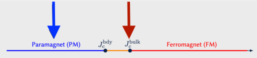
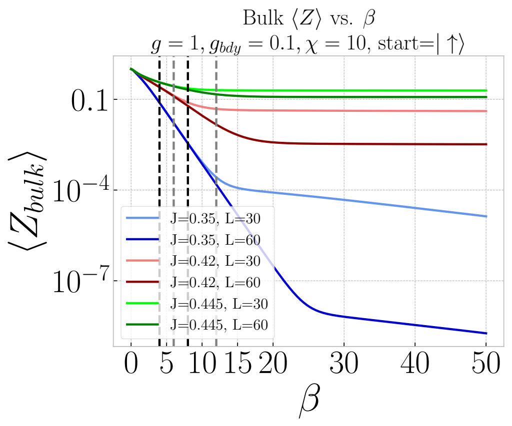
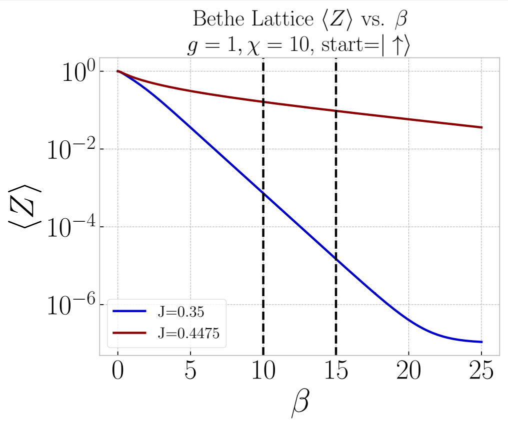
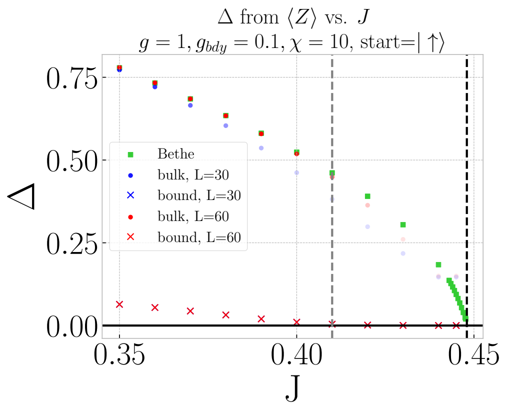
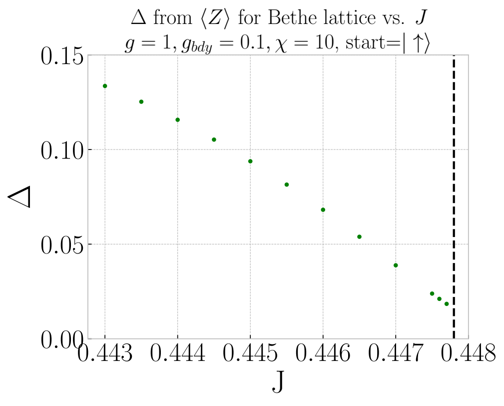










- Gapless at boundary transition, Gapped at boundary transition(?)

- Hastings-type bounds *[arXiv/1008.5137]*: Gap $\Delta \implies \langle AB\rangle_c \leq O(e^{-\Delta})$
- Technical subtlety; bound on:

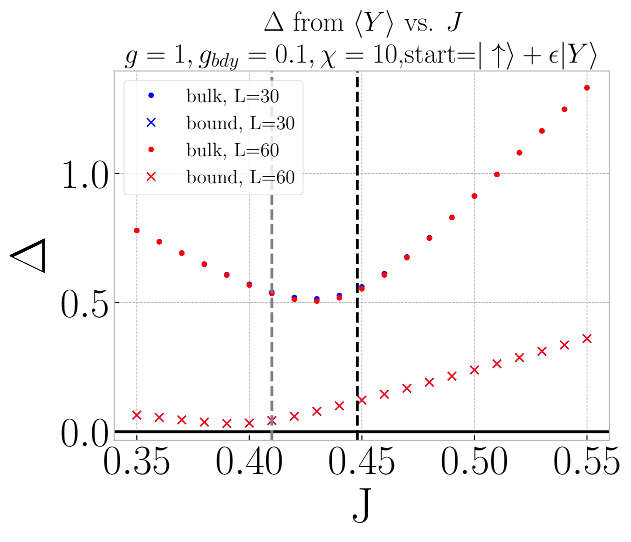
$$\langle AB\rangle_c = \langle\psi_0|\, AB\, |\psi_0\rangle - \langle\psi_0|\, AP_0B\, |\psi_0\rangle$$

with:

$$P_0 = \sum_a |\psi_0^a\rangle\langle\psi_0^a|$$

- With P_0 terms, the decay looks exponential (theorem is safe!) but the "physical" correlator decays algebraically.

Results - Spectrum from Dynamic Correlations (Y)



Motivation - Measurement-Based State Prep

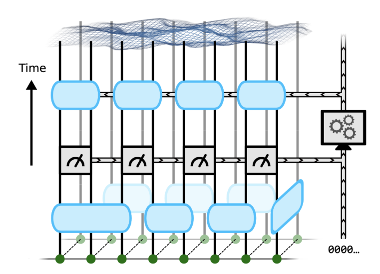
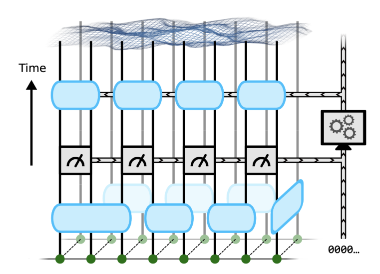


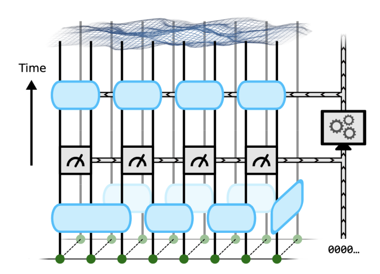
Image credit: PRX Quantum 3, 040337



- LRE states in constant time (GHZ *[Briegel & Raussendorf, PRL (2001)]*, Toric code *[Raussendorf, Brayvi & Hastings, PRA (2005)]*,…)

Image credit: PRX Quantum 3, 040337

Motivation - Measurement-Based State Prep



- LRE states in constant time (GHZ *[Briegel & Raussendorf, PRL (2001)]*, Toric code *[Raussendorf, Brayvi & Hastings, PRA (2005)]*,...)
- Extended to non-stabilizer states, variable correlation length states, e.g.:

$$|\Psi_\beta\rangle = \exp(\beta H_{\mathrm{TC}})\,|0\rangle^{\otimes N} \sim \exp\left(\beta \prod_s X_s\right)|0\rangle^{\otimes N}$$

(many works... *[arXiv/2404.16083, 2404.16360, 2404.16753, 2404.17087, 2405.09615]*)

Image credit: PRX Quantum 3, 040337

- What about critical states? E.g. (via map to Ising):

$$|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij\rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$$

- The landscape:

- What about critical states? E.g. (via map to Ising):

$$|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$$

- The landscape:
  - 1-D: No transition, but easily preparable *[Sahay and Veressen, PRX Quantum (2025)]*

Motivation - Preparing Critical States

- What about critical states? E.g. (via map to Ising):

$$|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$$

- The landscape:
  - 1-D: No transition, but easily preparable *[Sahay and Veressen, PRX Quantum (2025)]*
  - 2-D Square: Transition, but not easily preparable *[Zhu et al, PRL (2023)]*

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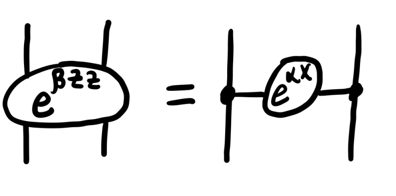
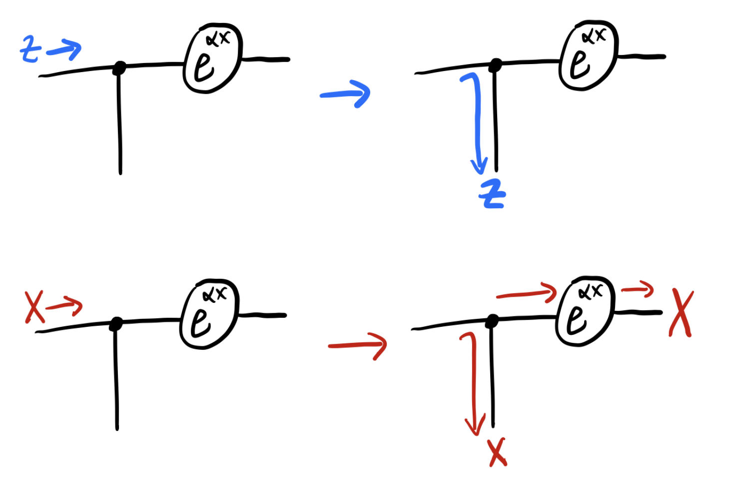
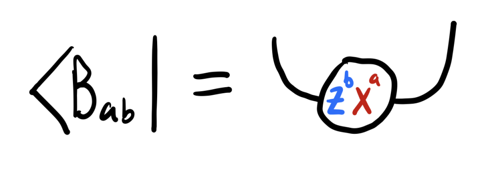
- The landscape:
  - 1-D: No transition, but easily preparable *[Sahay and Veressen, PRX Quantum (2025)]*
  - 2-D Square: Transition, but not easily preparable *[Zhu et al, PRL (2023)]*
  - Trees: Transition to “boundary sensitive” phase *[Eggarter, PRB (1974), Wang et al. PRB (2025)]* + preparable!

Techniques - Preparing $\exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$ (Step 1)



- With $\tanh(\beta) = e^{-2\alpha}$; both are $\text{diag}(e^{\beta}, e^{-\beta}, e^{-\beta}, e^{\beta})$.


z
e^{αx}
z
e^{αx}
X
e^{αx}
X
e^{αx}
X

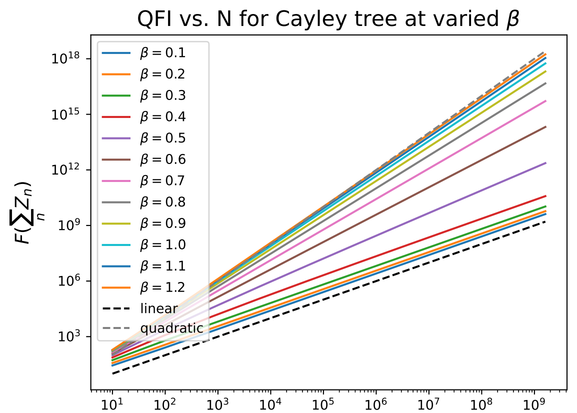


- 3/4-qubit spiders connected via Bell measurements
- Push errors to boundary, then single layer of cleanup!

Observing the (unconventional) phase transition

- Predicted transition at $\beta_c = \frac{1}{2}\ln\left(\frac{q}{q-2}\right) \approx 0.55$ can be observed from the QFI of the state:

$$\text{QFI}_\beta\left(\sum_i Z_i\right) = \left\langle \sum_{ij} Z_i Z_j \right\rangle_\beta - \left\langle \sum_i Z_i \right\rangle_\beta^2$$



Conclusions/Highlights

- (Simple) spin systems in hyperbolic space can host unintuitive fundamental phenomena
    - Boundary sensitive phase diagrams
    - New interplays of gaps and correlations!
- Protocols for (elusive) efficient construction of critical states!