Quantum Phases of Matter in Hyperbolic Space

Leibniz University Hannover Talk

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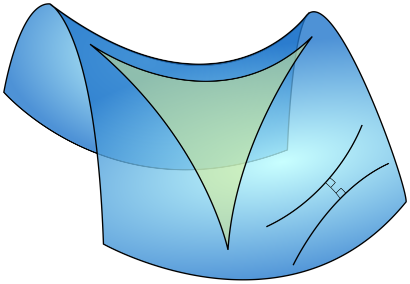
October 1, 2025

Motivation - What is hyperbolic space?



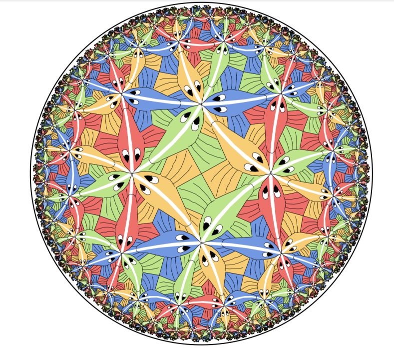
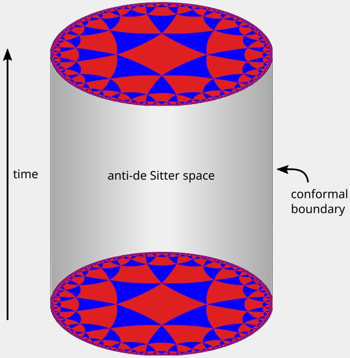
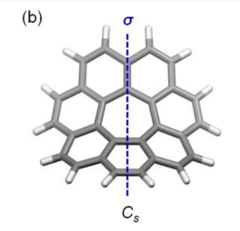
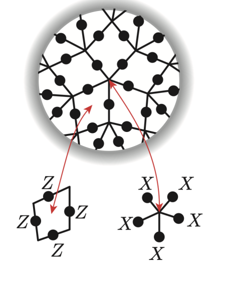


Image Credits: Wikipedia; D. Dunham (Transformation of Hyperbolic Escher Patterns)

Motivation - General Survey







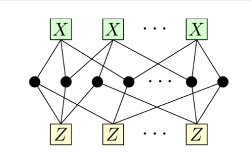
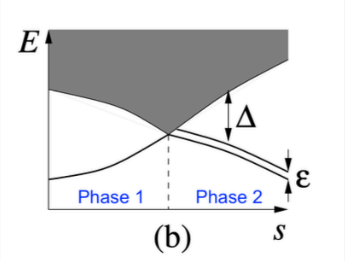
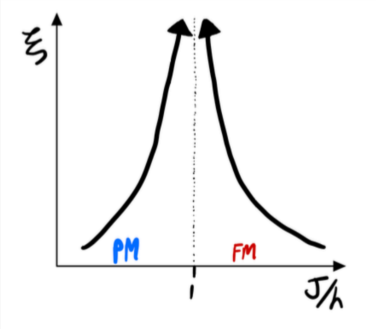


Image Credits: Wikipedia; Org. Lett. 2017, 19, 9, 2246-2249; arXiv:1703.00590; Quantum 5, 585 (2021)

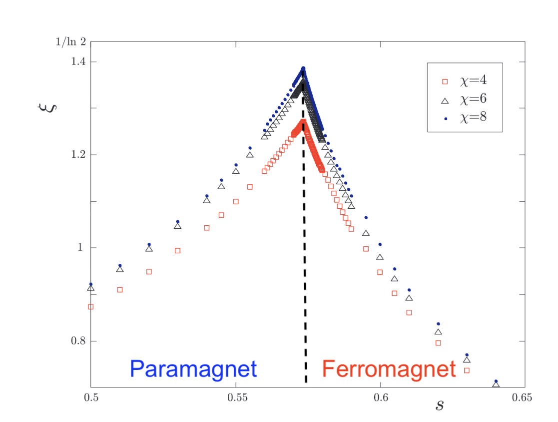




- Euclidean: Established relationships between adiabaticity/phases/gaps/correlation lengths.

Image Credit: Phys. Rev. B 82, 155138

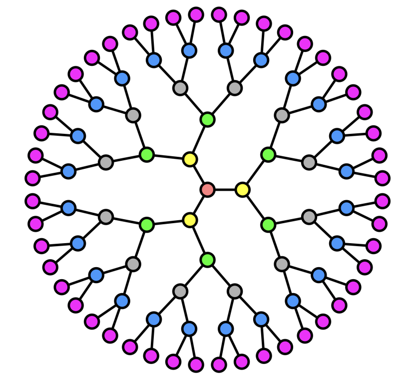
Motivation - Specific Questions



- Hyperbolic lattices: Intuitions called into question...
  - Indications of strangeness: No Goldstone bosons on Bethe lattice, non-divergent correlation lengths...
- Efficient preparability of states?

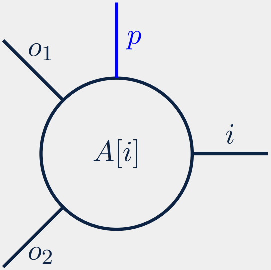
Image Credit: Phys. Rev. B 77, 214431

Setting - TFIM on Cayley Tree/Bethe Lattice

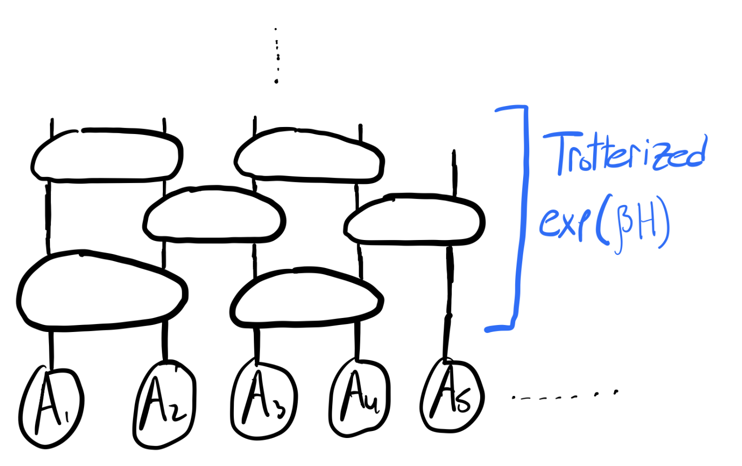


$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_{i \text{ bulk}} X_i - g_{\text{bdy}} \sum_{i \text{ boundary}} X_i$$

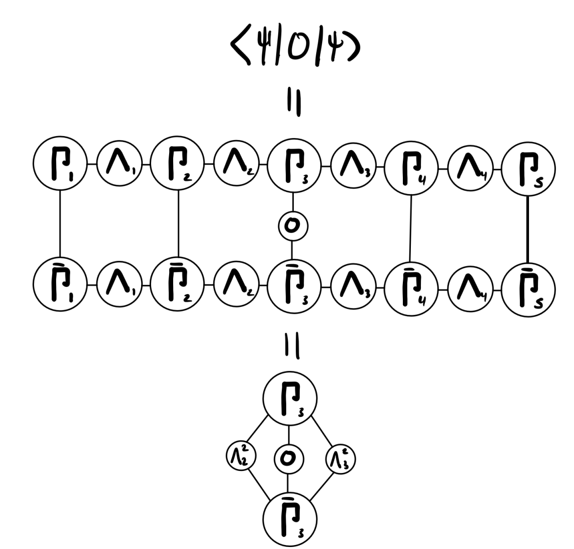
Image Credit: arXiv:1406.2819



- Rotational symmetry $\implies$ $O(L)$ tensors for L rings ($\sim 2^L$(!) qubits!)

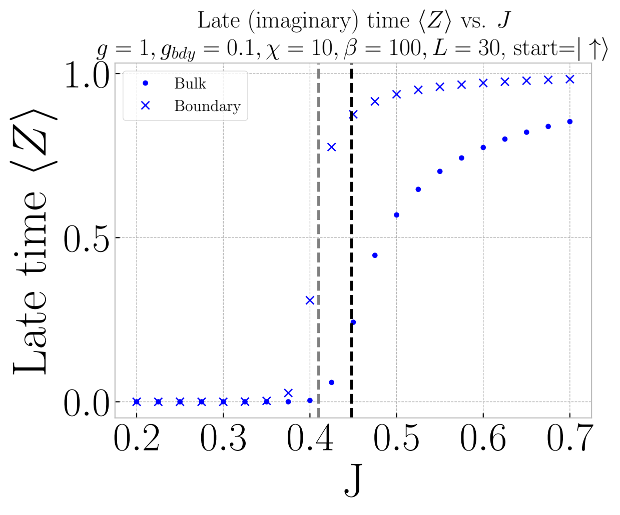


- Trotterized $\exp(\beta H) / \exp(itH)$ can be applied to simulate time evolution

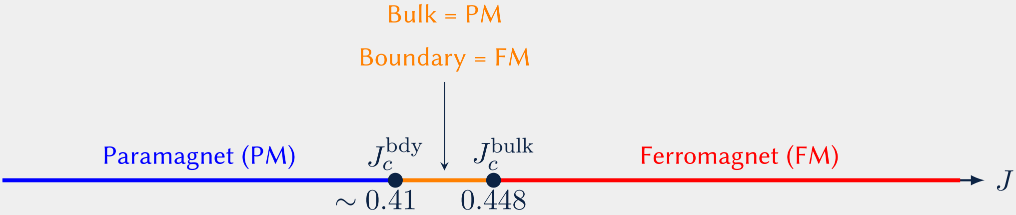


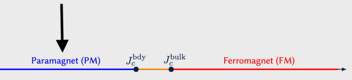
- Canonical form makes computing local expectation values efficient/stable.

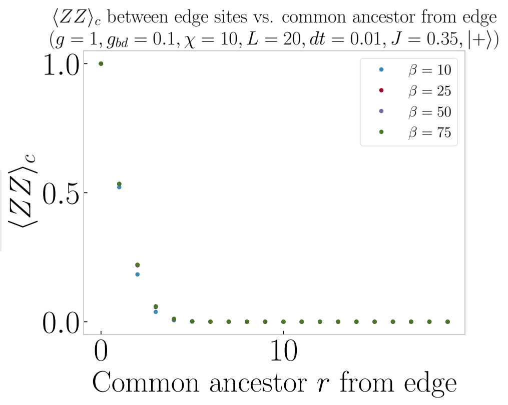
Results - Phase Diagram



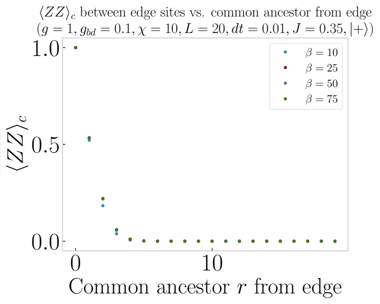
Results - Phase Diagram

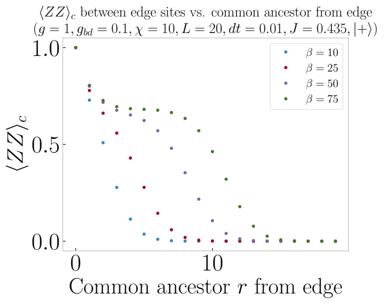


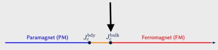

Paramagnet (PM)
J_c^{bdy}
J_c^{bulk}
Ferromagnet (FM)

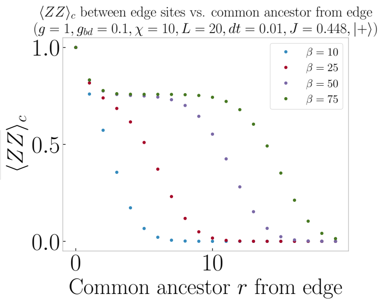

$\langle ZZ \rangle_c$ between edge sites vs. common ancestor from edge
$(g = 1, g_{bd} = 0.1, \chi = 10, L = 20, dt = 0.01, J = 0.35, |+\rangle)$
$\beta = 10$
$\beta = 25$
$\beta = 50$
$\beta = 75$
$\langle ZZ \rangle_c$
1.0
0.5
0.0
0
10
Common ancestor r from edge

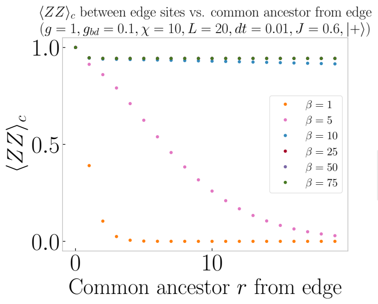
Results - Static Spatial Correlations (Cat State Growth)



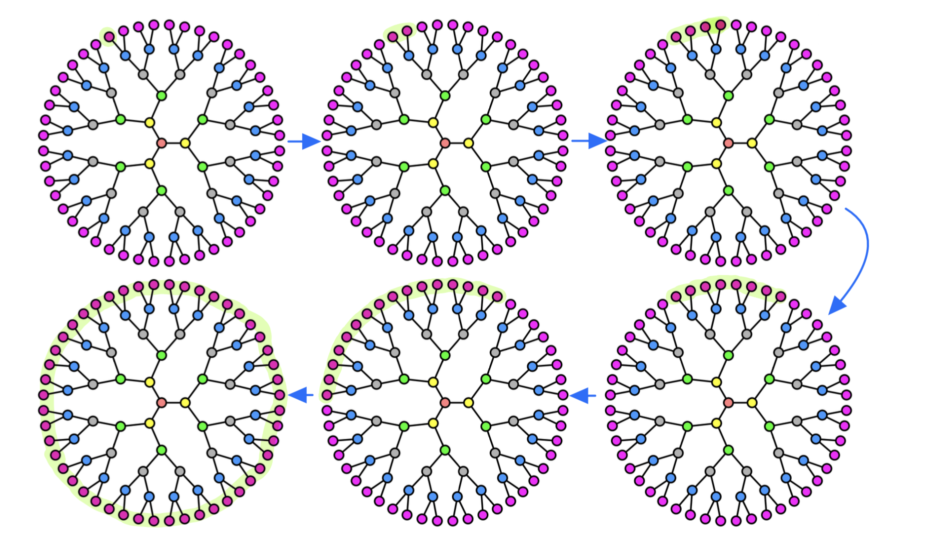




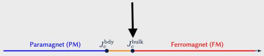


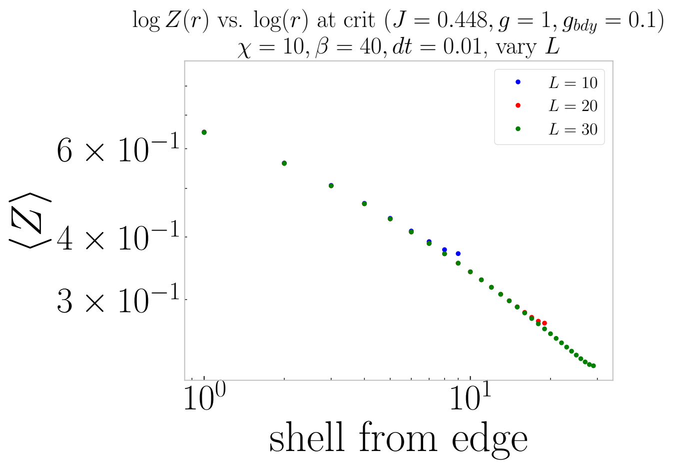


Results - Static Spatial Correlations (Cat State Growth)



Results - Static Spatial Correlations (Algebraic Decay)





Results - Spectrum from Dynamic Correlations (Z)

$$\langle\uparrow| e^{-\beta H} Z e^{-\beta H} |\uparrow\rangle = \sum_{nm} e^{-\beta(E_n+E_m)} \langle\uparrow|m\rangle \langle n|\uparrow\rangle \langle m| Z |n\rangle \approx e^{-\beta(0+\Delta)} \langle\uparrow|0\rangle \langle 1|\uparrow\rangle \langle 0| Z |1\rangle$$

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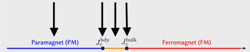
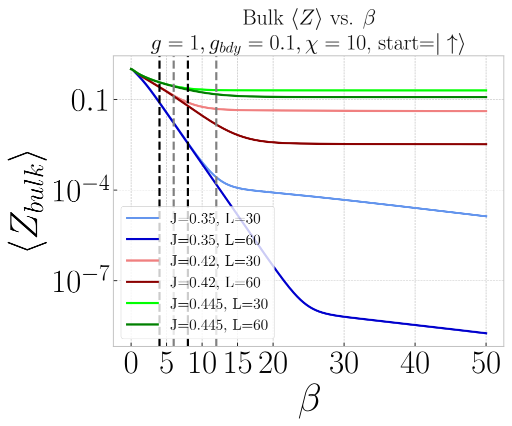
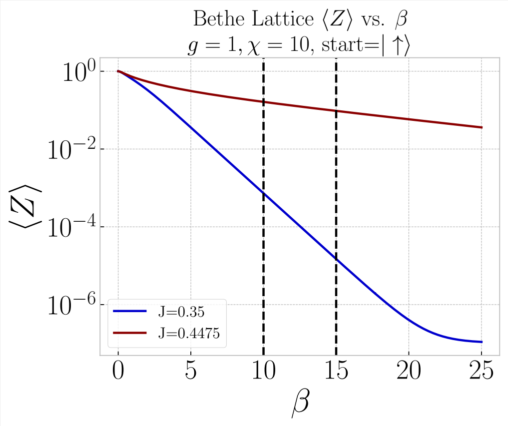
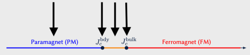
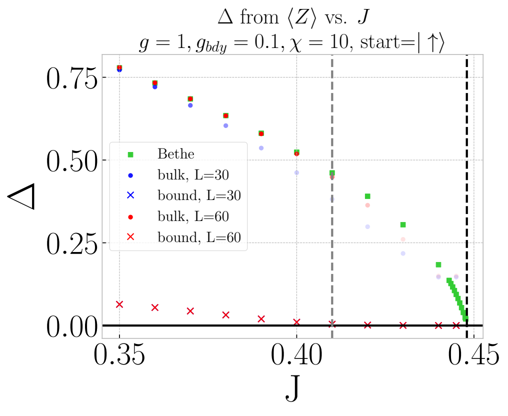
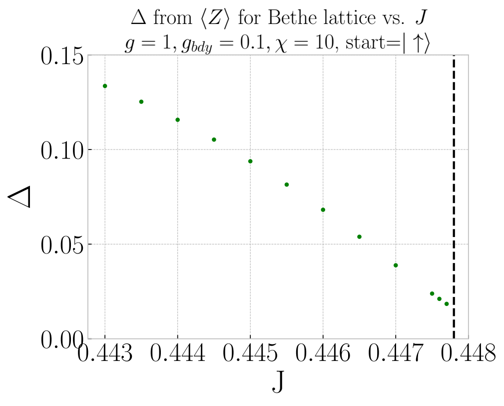






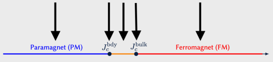


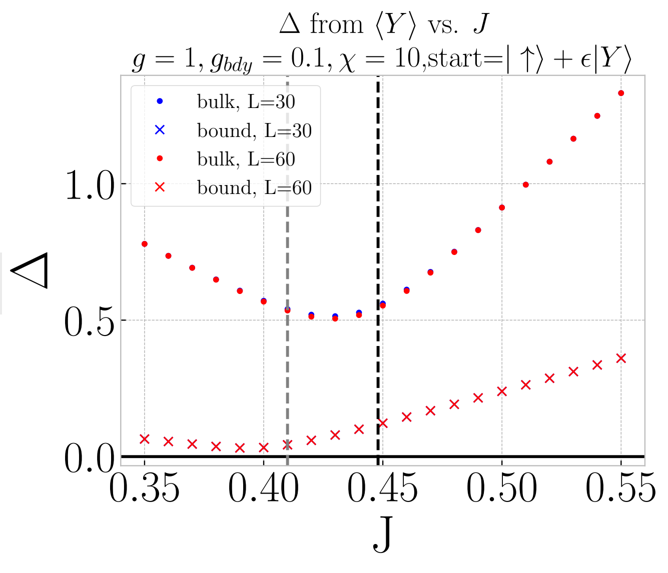


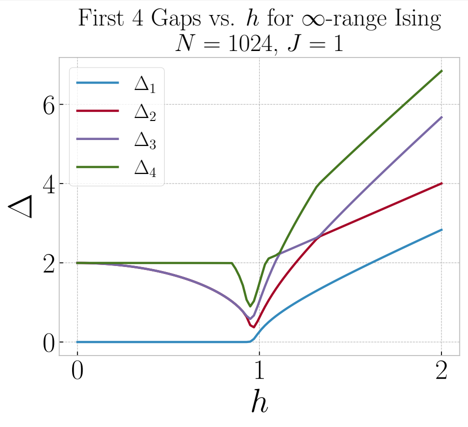


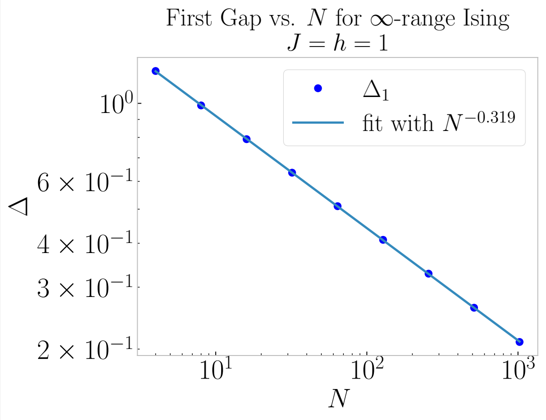
- Gapless at boundary transition, Gapped at boundary transition(?)

Results - Spectrum from Dynamic Correlations (Y)





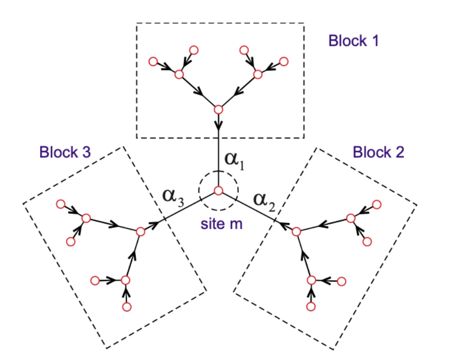




$$H = -\frac{J}{N} \sum_{i,j} Z_i Z_j - h \sum_i X_i = -\frac{J}{2N} \left(\sum_i Z_i \right)^2 - h \sum_i X_i$$

- Exact diagonalization of "superspin"

Next Steps - DMRG on a tree



- Does imaginary time evolution give the true GS? Check with DMRG.

Image Credit: Phys. Rev. B 82, 205105

Next Steps - Measurement-Based State Prep

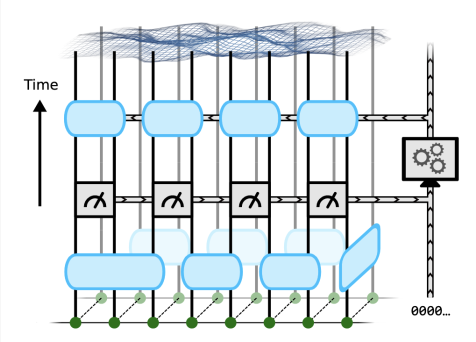
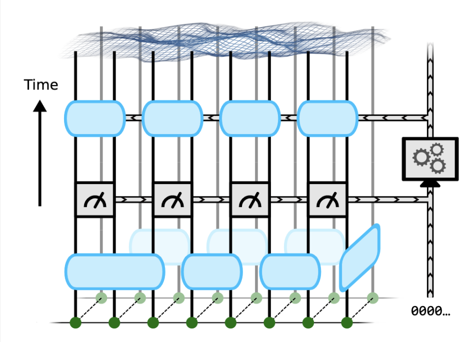


Image credit: PRX Quantum 3, 040337



- LRE states in constant time (GHZ, Toric code…)
- Extended to non-stabilizer states, variable correlation length states, e.g.:

$$|\Psi_\beta\rangle = \exp(\beta H_{\text{TC}})\,|0\rangle^{\otimes N} \sim \exp\left(\beta \prod_s X_s\right)|0\rangle^{\otimes N}$$

Image credit: PRX Quantum 3, 040337

Next Steps - Measurement-Based State Prep on a Tree

- Efficient preparation of quantum critical states? E.g. $|\Phi_\beta\rangle = \exp\Big(\beta \sum_{\langle ij \rangle} Z_i Z_j\Big) |+\rangle^{\otimes N}$

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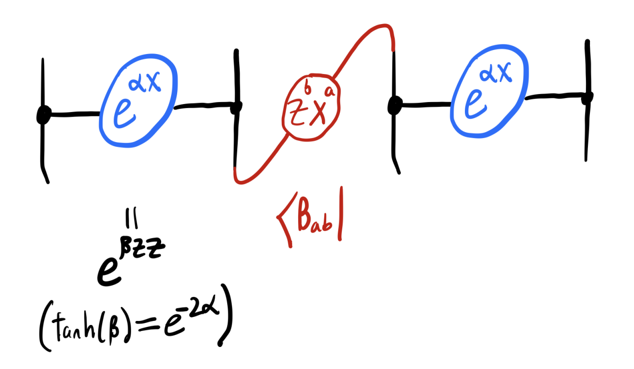

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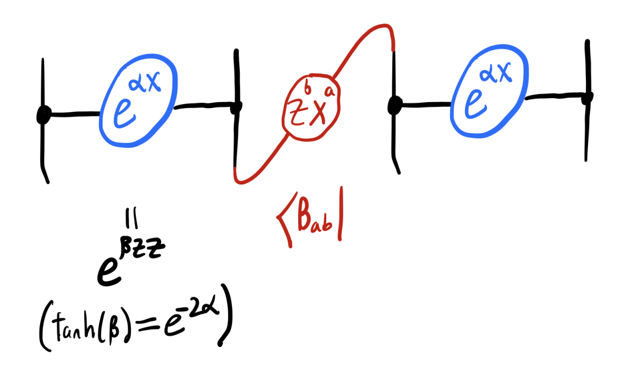

- The landscape:
  - 1-D: Preparable (but no transition)
  - 2-D Square: Not preparable (frustration)
  - Hyperbolic trees (sweet spot?)

Conclusions

- Hyperbolic lattices are unintuitive, but useful!
- Boundary-sensitive phase diagram
- Energy/Correlations of bulk and boundary
- Lack of loops:
  - Efficient simulation (tensor networks)
  - Efficient measurement-based state prep(?)