

QUANTUM PHASES OF MATTER IN HYPERBOLIC SPACE

LEIBNIZ UNIVERSITY HANNOVER TALK

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MOTIVATION - WHAT IS HYPERBOLIC SPACE?

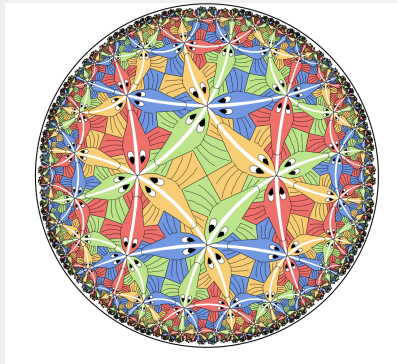
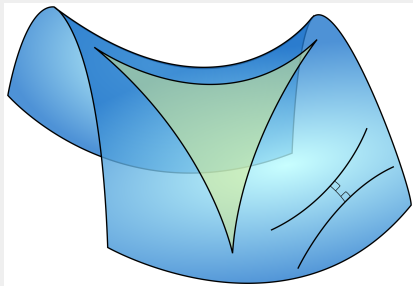


Image Credits: Wikipedia; D. Dunham (Transformation of Hyperbolic Escher Patterns)

MOTIVATION - GENERAL SURVEY

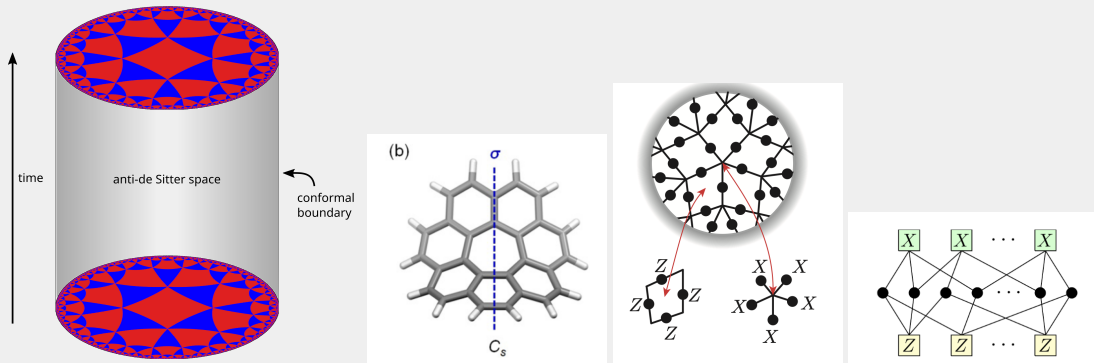
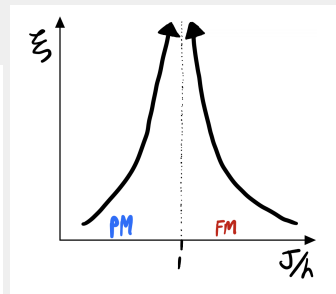
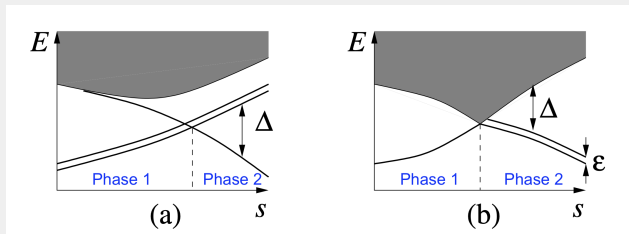


Image Credits: Wikipedia; Org. Lett. 2017, 19, 9, 2246-2249; arXiv:1703.00590; Quantum 5, 585 (2021)

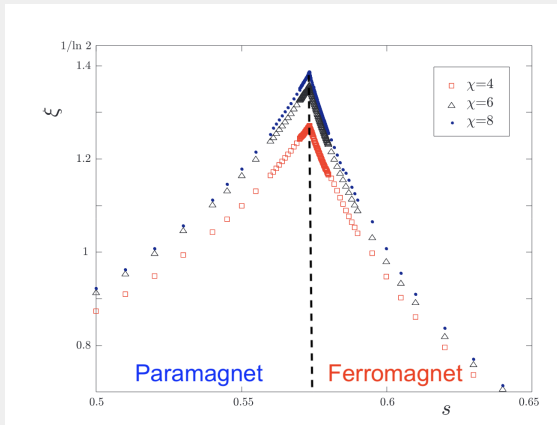
MOTIVATION - SPECIFIC QUESTIONS



- Euclidean: Established relationships between adiabaticity/phases/gaps/correlation lengths.

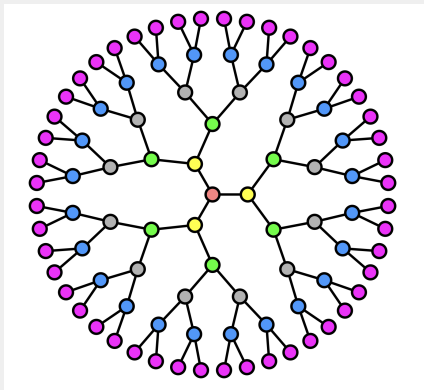
Image Credit: Phys. Rev. B 82, 155138

MOTIVATION - SPECIFIC QUESTIONS



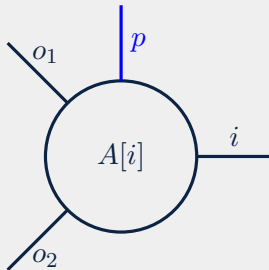
- Hyperbolic lattices: Intuitions called into question...
 - ▶ Indications of strangeness: No Goldstone bosons on Bethe lattice, non-divergent correlation lengths...
- Efficient preparability of states?

SETTING - TFIM ON CAYLEY TREE/BETHE LATTICE

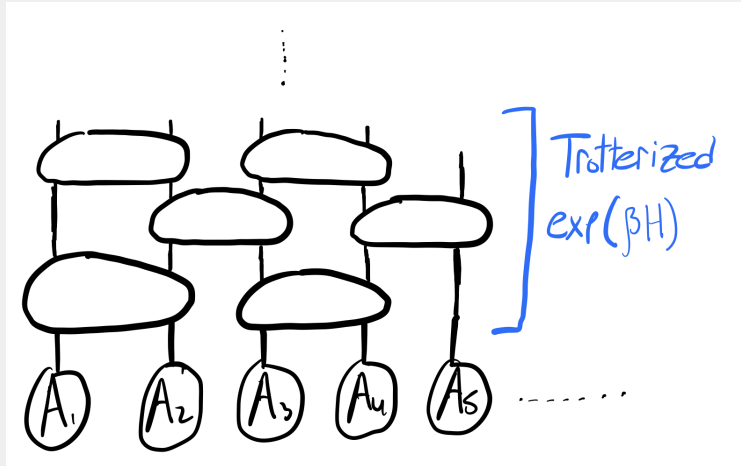


$$H = -J \sum_{\langle ij \rangle} Z_i Z_j - g \sum_{i \text{ bulk}} X_i - g_{\text{bdy}} \sum_{i \text{ boundary}} X_i$$

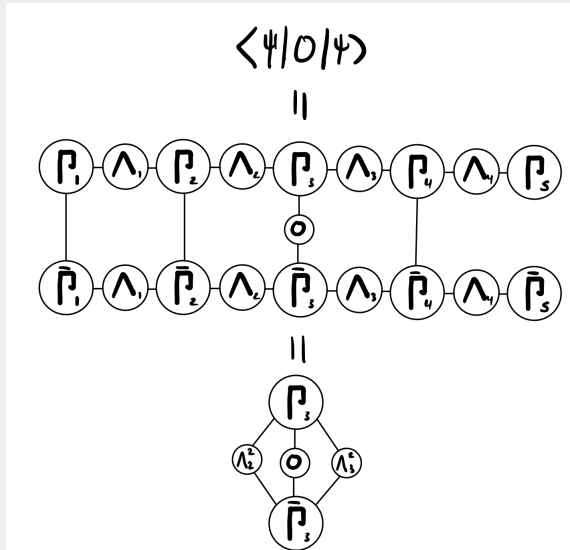
Image Credit: arXiv:1406.2819



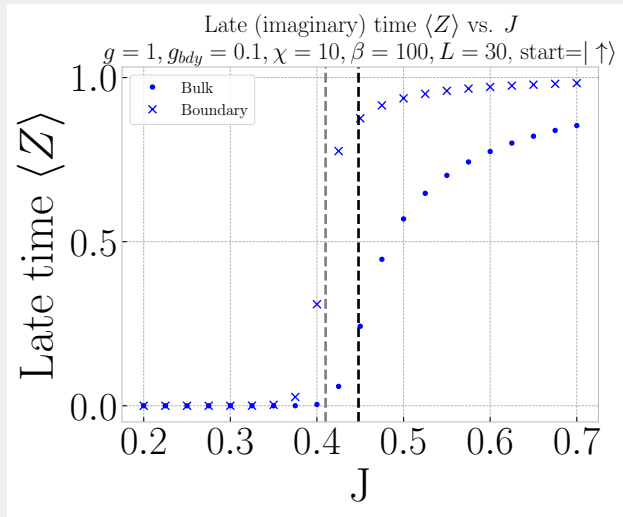
- Rotational symmetry $\implies O(L)$ tensors for L rings ($\sim 2^L$ (!) qubits!)



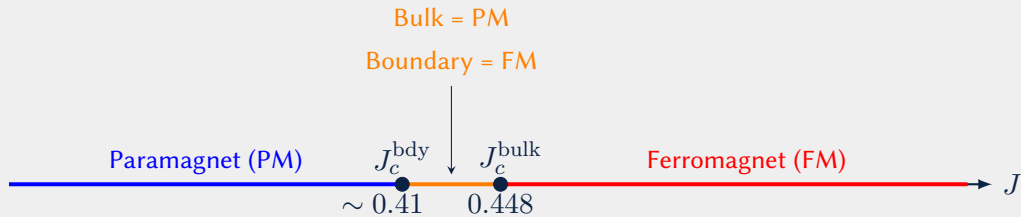
- Trotterized $\exp(\beta H) / \exp(itH)$ can be applied to simulate time evolution



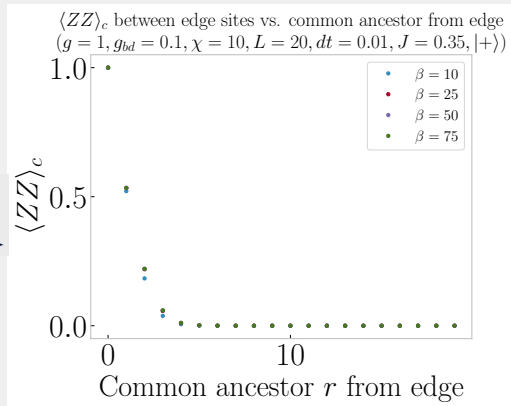
- Canonical form makes computing local expectation values efficient/stable.



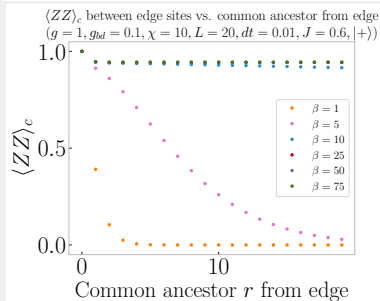
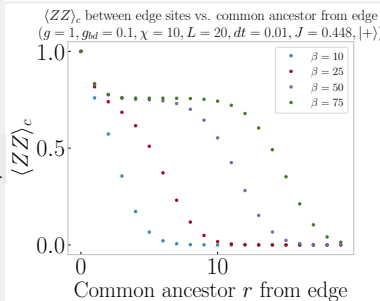
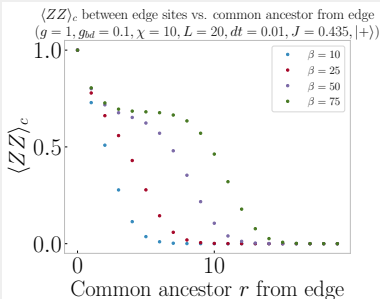
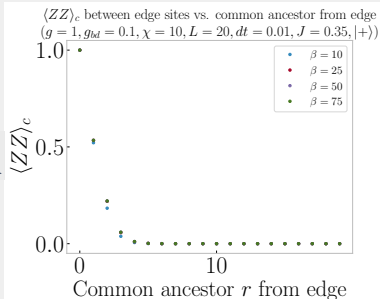
RESULTS - PHASE DIAGRAM



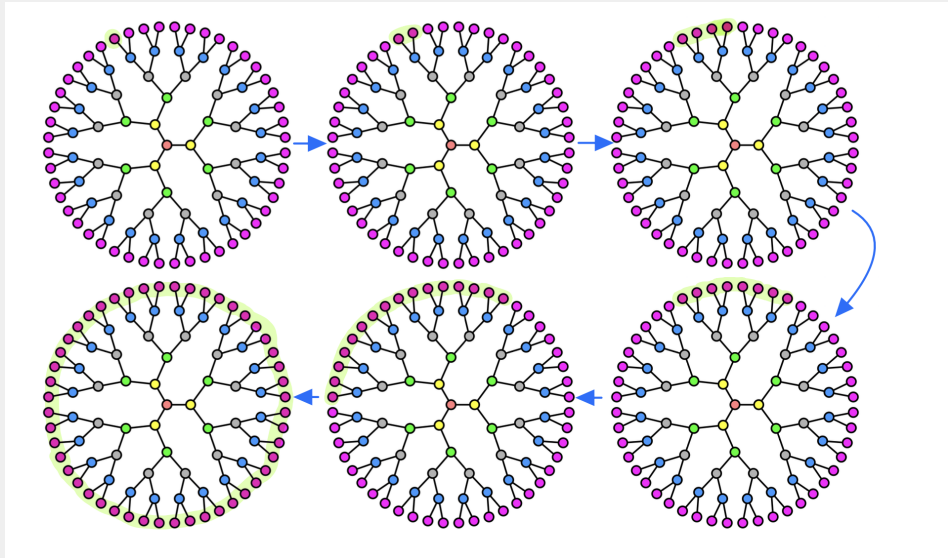
RESULTS - STATIC SPATIAL CORRELATIONS (CAT STATE GROWTH)



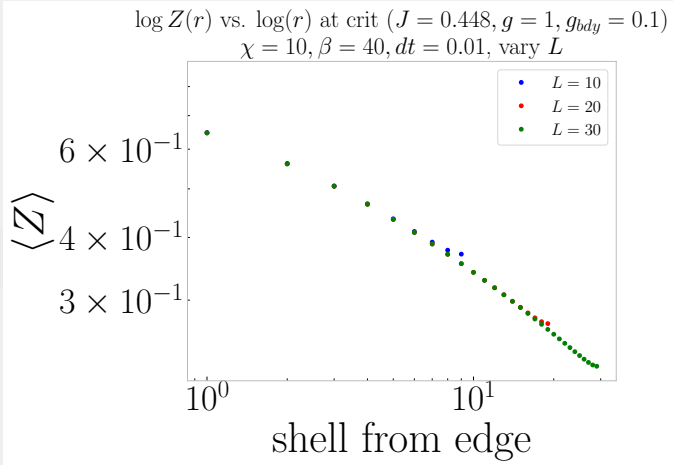
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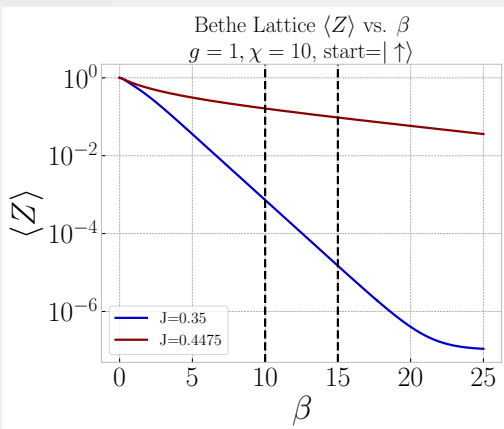
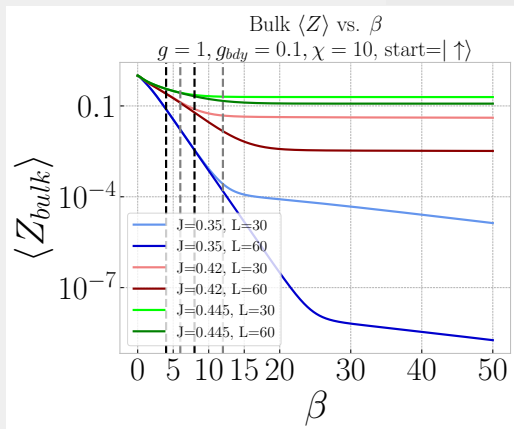
RESULTS - STATIC SPATIAL CORRELATIONS (ALGEBRAIC DECAY)



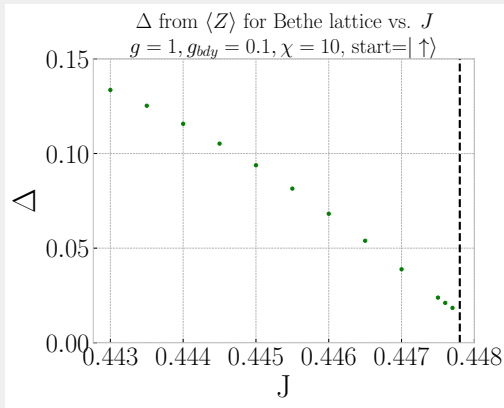
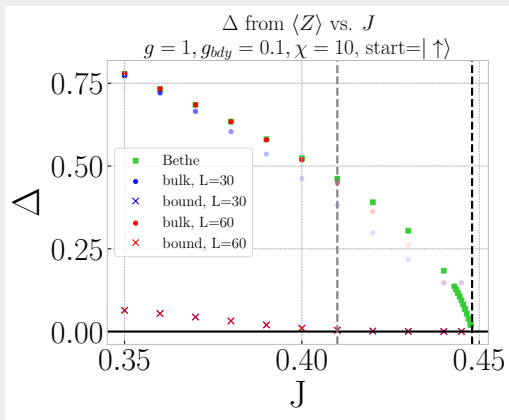
$$\langle \uparrow | e^{-\beta H} Z e^{-\beta H} | \uparrow \rangle = \sum_{nm} e^{-\beta(E_n + E_m)} \langle \uparrow | m \rangle \langle n | \uparrow \rangle \langle m | Z | n \rangle \approx e^{-\beta(0 + \Delta)} \langle \uparrow | 0 \rangle \langle 1 | \uparrow \rangle \langle 0 | Z | 1 \rangle$$

RESULTS - SPECTRUM FROM DYNAMIC CORRELATIONS (Z)

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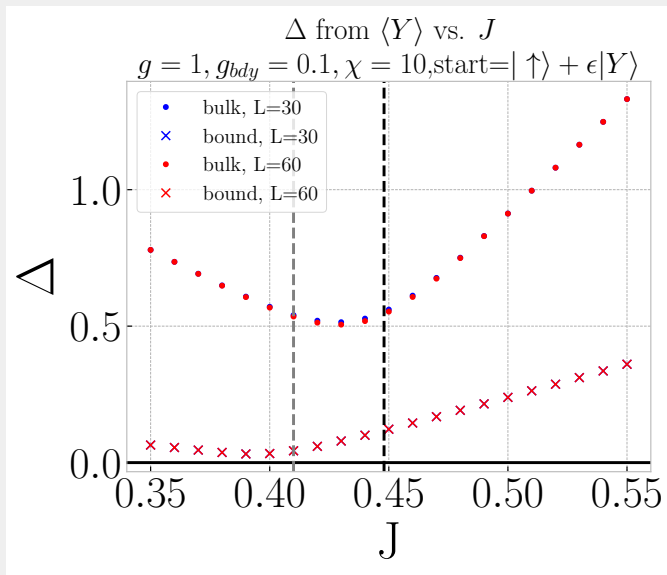


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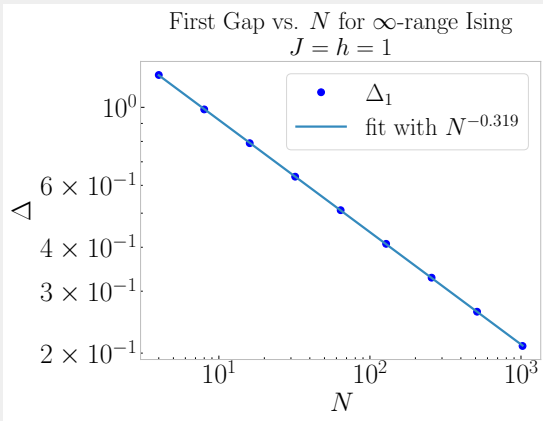
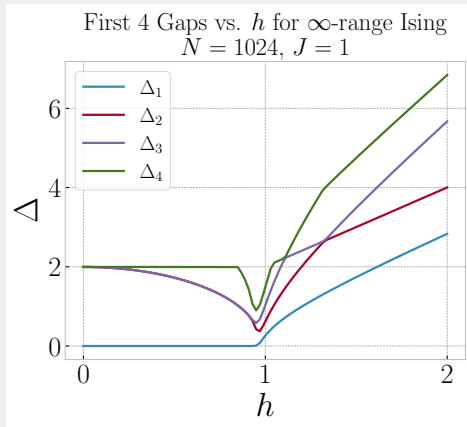


- Gapless at boundary transition, Gapped at boundary transition(?)

RESULTS - SPECTRUM FROM DYNAMIC CORRELATIONS (Y)



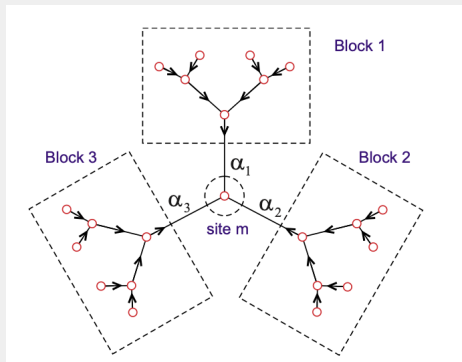
RESULTS - COMPARISON TO INFINITE-RANGE CASE



$$H = -\frac{J}{N} \sum_{i,j} Z_i Z_j - h \sum_i X_i = -\frac{J}{2N} \left(\sum_i Z_i \right)^2 - h \sum_i X_i$$

- Exact diagonalization of “superspin”

NEXT STEPS - DMRG ON A TREE



- Does imaginary time evolution give the true GS? Check with DMRG.

NEXT STEPS - MEASUREMENT-BASED STATE PREP

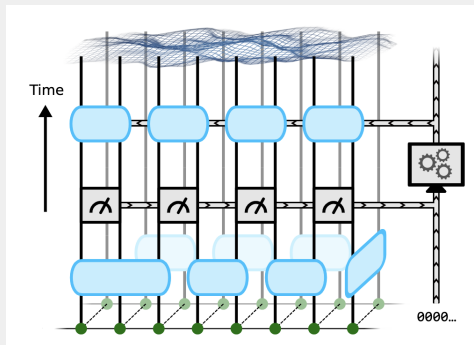
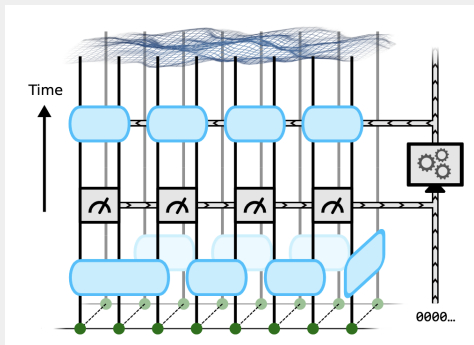


Image credit: PRX Quantum 3, 040337

NEXT STEPS - MEASUREMENT-BASED STATE PREP



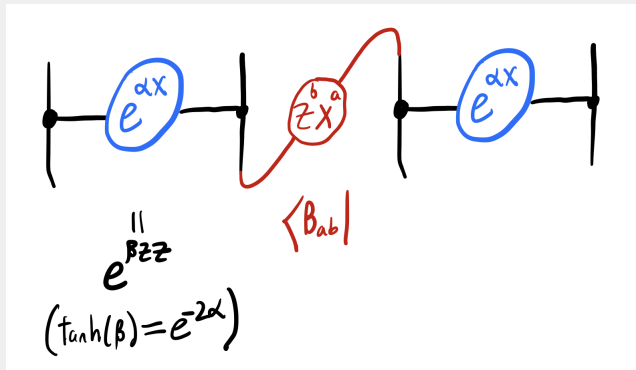
- LRE states in constant time (GHZ, Toric code...)
- Extended to non-stabilizer states, variable correlation length states, e.g.:

$$|\Psi_\beta\rangle = \exp(\beta H_{\text{TC}}) |0\rangle^{\otimes N} \sim \exp\left(\beta \prod_s X_s\right) |0\rangle^{\otimes N}$$

- Efficient preparation of quantum critical states? E.g. $|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$

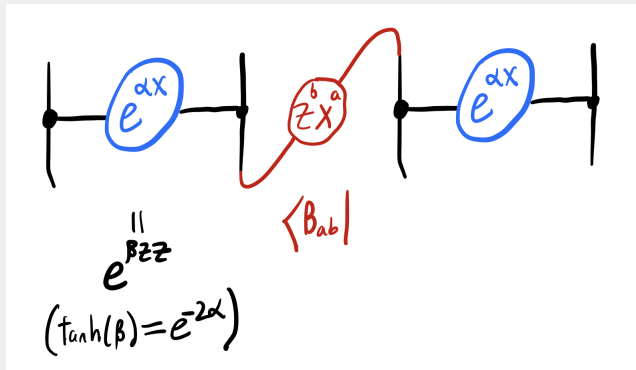
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NEXT STEPS - MEASUREMENT-BASED STATE PREP ON A TREE

- Efficient preparation of quantum critical states? E.g. $|\Phi_\beta\rangle = \exp\left(\beta \sum_{\langle ij \rangle} Z_i Z_j\right) |+\rangle^{\otimes N}$



- The landscape:
 - ▶ 1-D: Preparable (but no transition)
 - ▶ 2-D Square: Not preparable (frustration)
 - ▶ Hyperbolic trees (sweet spot?)

- Hyperbolic lattices are unintuitive, but useful!
- Boundary-sensitive phase diagram
- Energy/Correlations of bulk and boundary
- Lack of loops:
 - ▶ Efficient simulation (tensor networks)
 - ▶ Efficient measurement-based state prep(?)