Quantifying resource states and efficient regimes of MBQC on a superconducting processor

UBC Group Meeting

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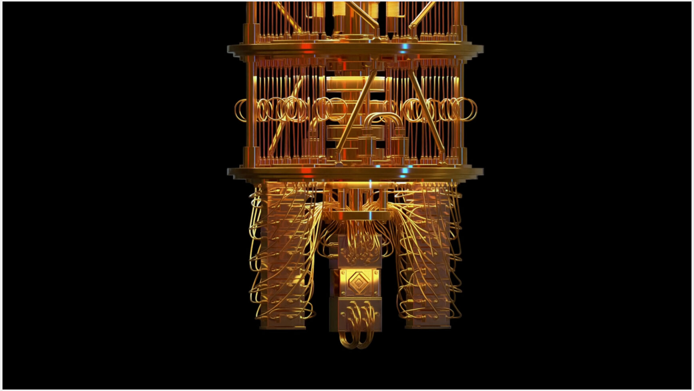
July 16, 2024

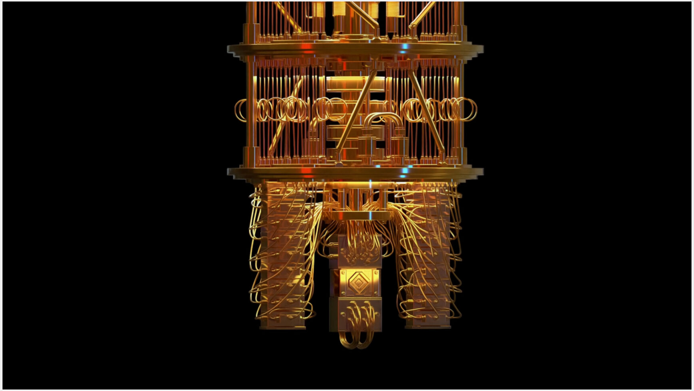
Motivating Questions



1. How do we characterize quantum computational speedup?

2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

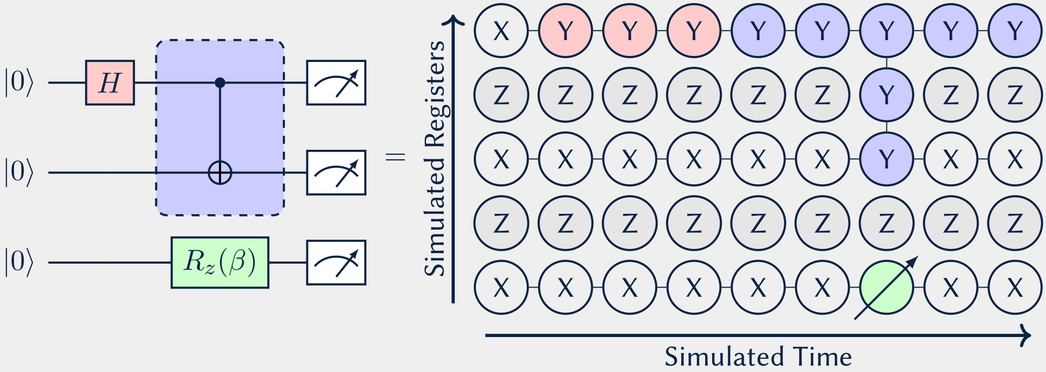
Image Credit: Quanta Magazine



1. How do we characterize quantum computational speedup?
   - One route - Measurement-Based Quantum Computing
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Fun playground for physicists!

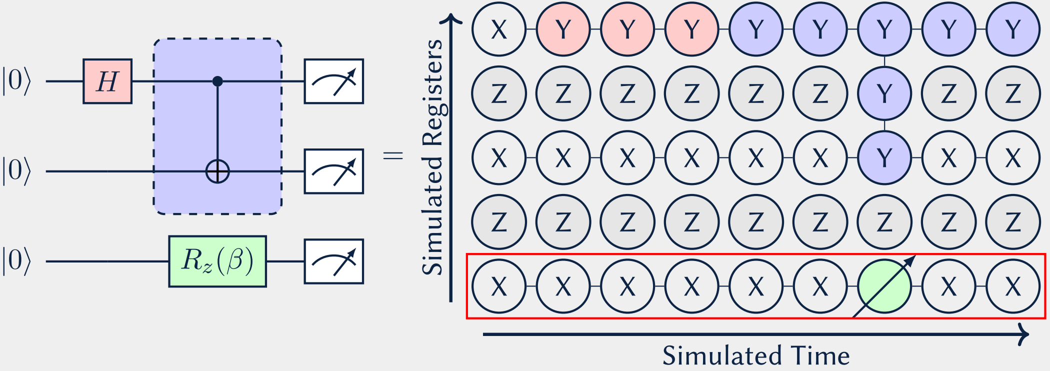
Image Credit: Quanta Magazine

A One-Slide Review of MBQC



	Circuit Model	MBQC
Initialization	$\|00\ldots0\rangle$	(Universal) resource state
Evolution	Unitary Gates	(Adaptive) singe-qubit measurements

A One-Slide Review of MBQC



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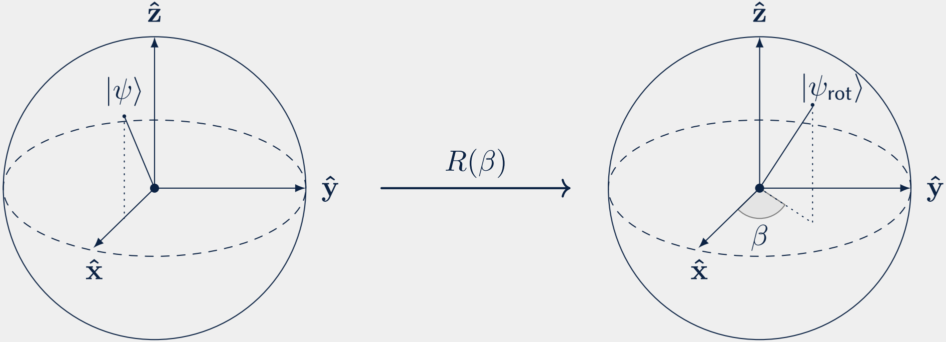
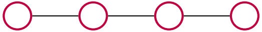


Ability to perform arbitrary single qubit unitaries (rotations) with high fidelity.

1D Resource States - 2 Examples and Interpolation

Universal Resource: Cluster State $|C\rangle$



Ground state of
$H_{\text{cluster}} = -\sum_i Z_{i-1} X_i Z_{i+1}$

Useless Resource: Product State $|+\rangle^{\otimes N}$

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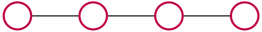

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Useless Resource: Product State $|+\rangle^{\otimes N}$

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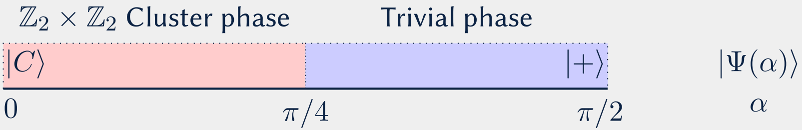
Computational order ground states $|\Psi(\alpha)\rangle$ of:

$$H(\alpha) = -\cos(\alpha) \sum_i Z_{i-1} X_i Z_{i+1} - \sin(\alpha) \sum_i X_i$$

1D Resource States - SPT Phases & Decoherence

Answer (for infinite systems):



- Computational power is uniform in symmetry-protected topological (SPT) phases.

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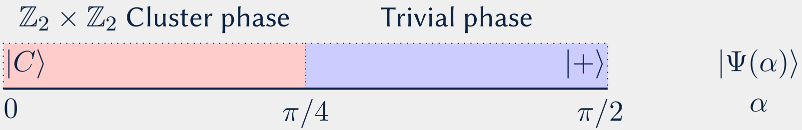

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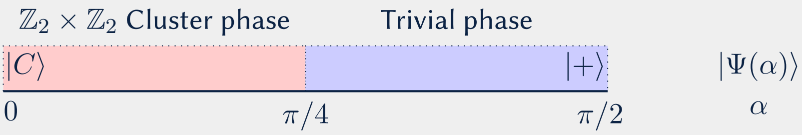

- Computational power is uniform in symmetry-protected topological (SPT) phases.
- The catch: Decoherence away from cluster state.
- Recently: New formalism for analyzing power in *finite* resource states.

- Desired logical rotation $\exp\left(-i\frac{\beta}{2}P\right)$ becomes a probabilistic channel:

$$\mathcal{V} = \frac{1+\nu}{2}\exp\left(-i\frac{\beta}{2}P\right) + \frac{1-\nu}{2}\exp\left(i\frac{\beta}{2}P\right)$$

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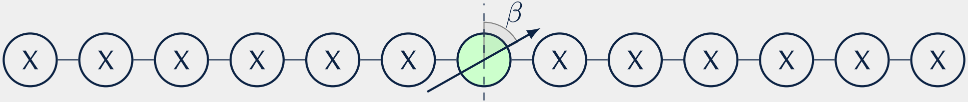
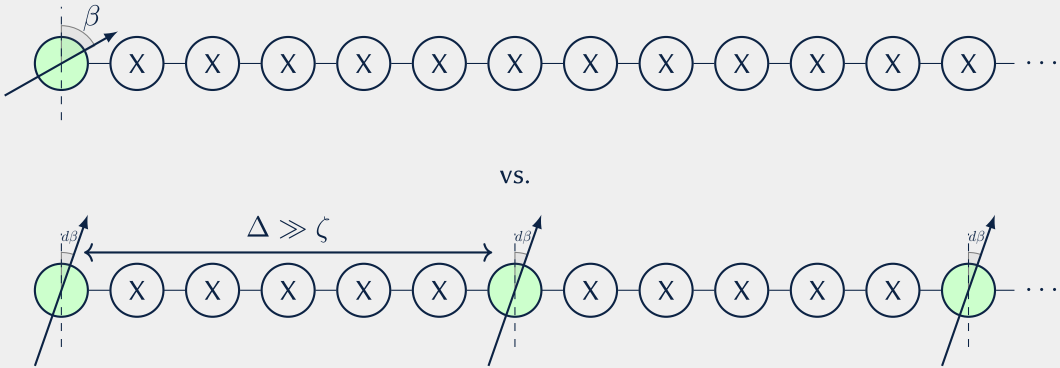
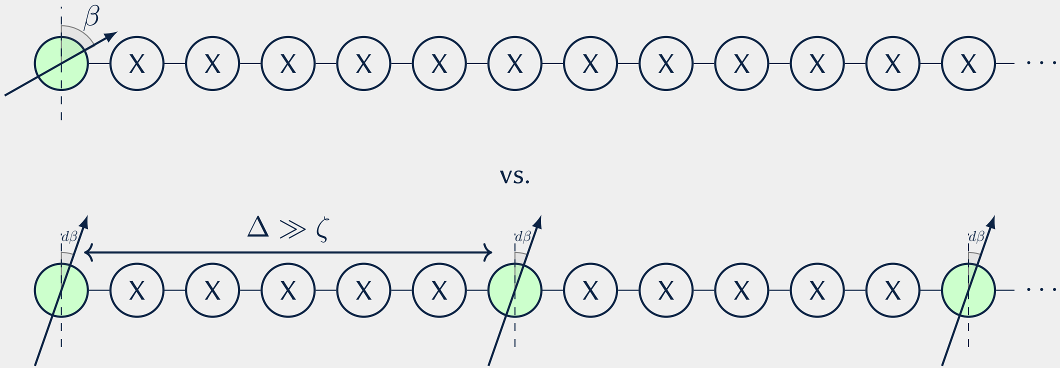


$$\sigma = \langle Z \quad X \quad I \quad X \quad I \quad X \quad Z \rangle$$

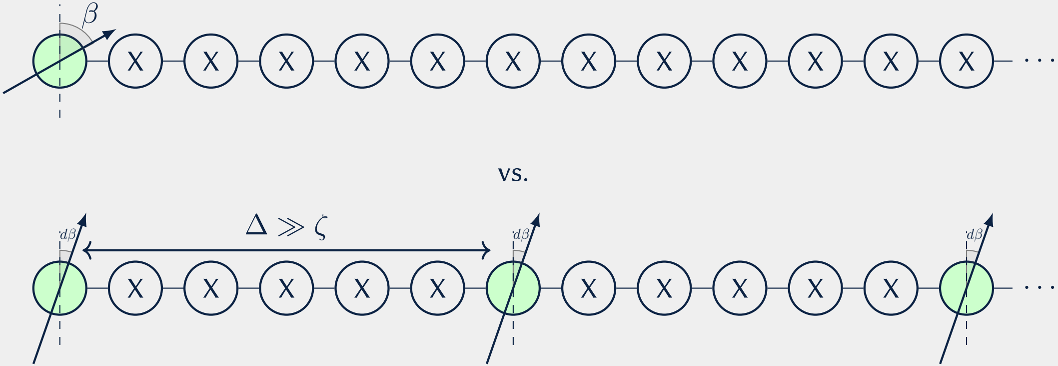


- Error is $O(\beta^2)$ - m subdivisions reduces error, with $\epsilon \propto (1-\sigma^2)\frac{\beta^2}{m}$ for small angles.

1D Resource States - Decoherence Management I



- Error is $O(\beta^2)$ - m subdivisions reduces error, with $\epsilon \propto (1-\sigma^2)\frac{\beta^2}{m}$ for small angles.
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as needed $\Rightarrow$ computational phases.



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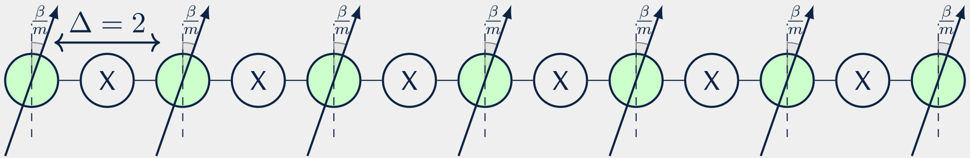
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as needed $\Rightarrow$ computational phases.
- Finite case: Tradeoff of rotation splitting and independence.



Optimal strategy: Assuming SOPs of state have convex decay, split as much as possible.

Proposed Experiments

1. Computational order = String order
2. Decoherence management I - Divide and conquer
3. Decoherence management II - The counterintuitive regime

Experiment 0 - Ground State Anstatz

Recall $H(\alpha)$:

$$H(\alpha) = -\cos(\alpha)\sum_i Z_{i-1}X_iZ_{i+1} - \sin(\alpha)\sum_i X_i$$

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We consider the following variational ansatz:

$$|\psi(\theta)\rangle = \bigotimes_{i=2}^{N-1} T_i(\theta)\,|\mathcal{C}\rangle = \bigotimes_{i=2}^{N-1}(\cos(\theta)I_i + \sin(\theta)X_i)\,|\mathcal{C}\rangle$$

Motivated by symmetry, perturbation theory, and efficiency.

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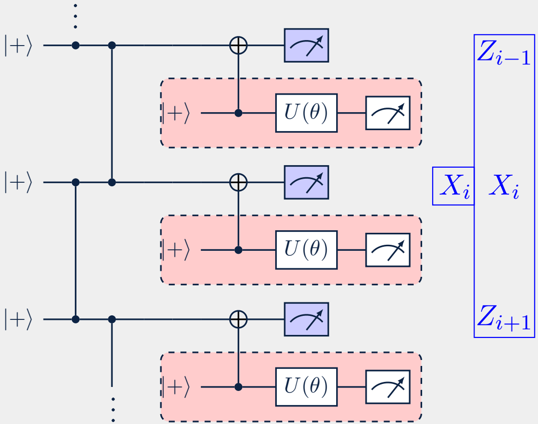
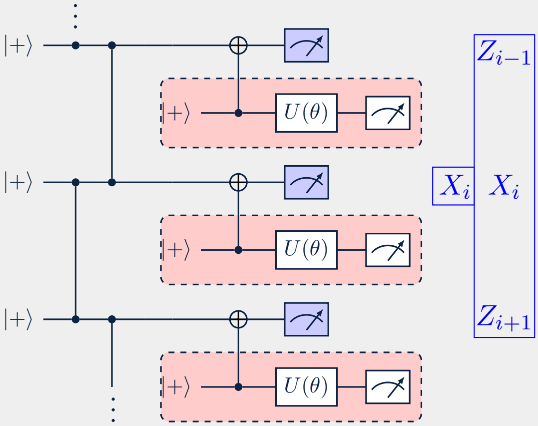
Motivated by symmetry, perturbation theory, and efficiency.
For a given value of α, find $|\psi(\theta)\rangle$ which minimizes:

$$\langle\psi(\theta)|\,H(\alpha)\,|\psi(\theta)\rangle = -\cos(\alpha)\sum_{i=1}^{N}\langle K_i = Z_{i-1}X_iZ_{i+1}\rangle_\theta - \sin(\alpha)\sum_{i=1}^{N}\langle X_i\rangle_\theta$$



Algorithm for finding $\langle X_i\rangle_\theta$ / $\langle K_i\rangle_\theta$:

1. Prepare the cluster ring $|\mathcal{C}_N\rangle$.
2. Probabilistically implement (non-unitary) $T_i(\theta) = \cos(\theta)I_i + \sin(\theta)X_i$ on each site.
3. Measure X_i or $K_i = Z_{i-1}X_iZ_{i+1}$ on the prepared state to obtain $\langle X_i\rangle_\theta$ / $\langle K_i\rangle_\theta$.



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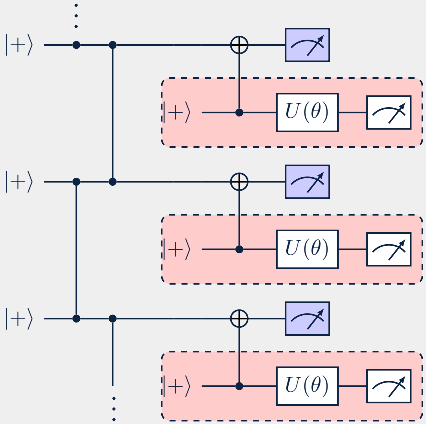
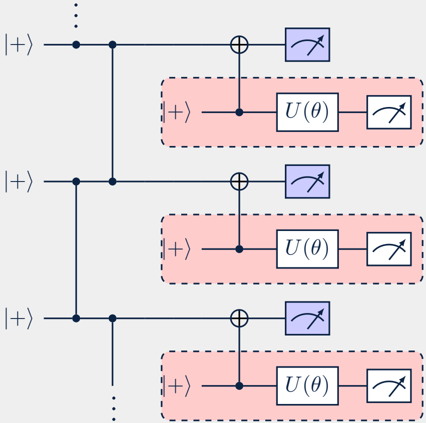
Then, various tricks with symmetry, half-teleportation, translation invariance...


$|+\rangle$
$|+\rangle$
$U(\theta)$
$|+\rangle$
$|+\rangle$
$U(\theta)$
$|+\rangle$
$|+\rangle$
$U(\theta)$

$$O(n)$$


$|+\rangle$
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$|+\rangle$
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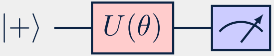
$|+\rangle$
$|+\rangle$
$U(\theta)$

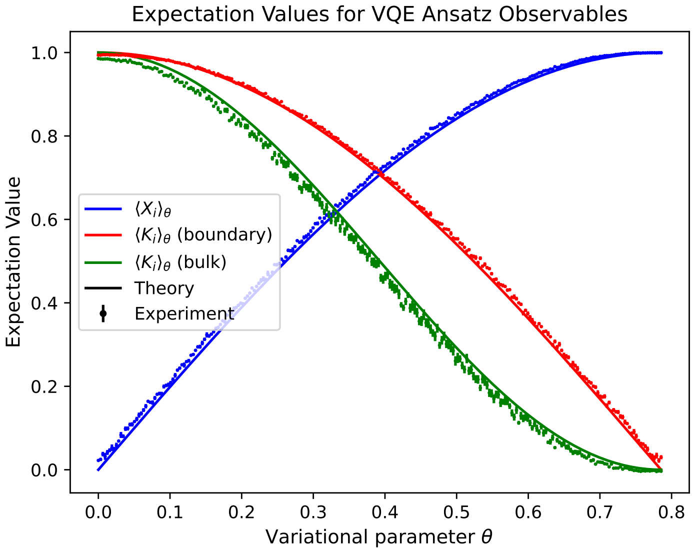
$\cong$


$|+\rangle$
$U(\theta)$

$O(1)!$

$O(n)$

Experiment 0 - State Preparation (Results)



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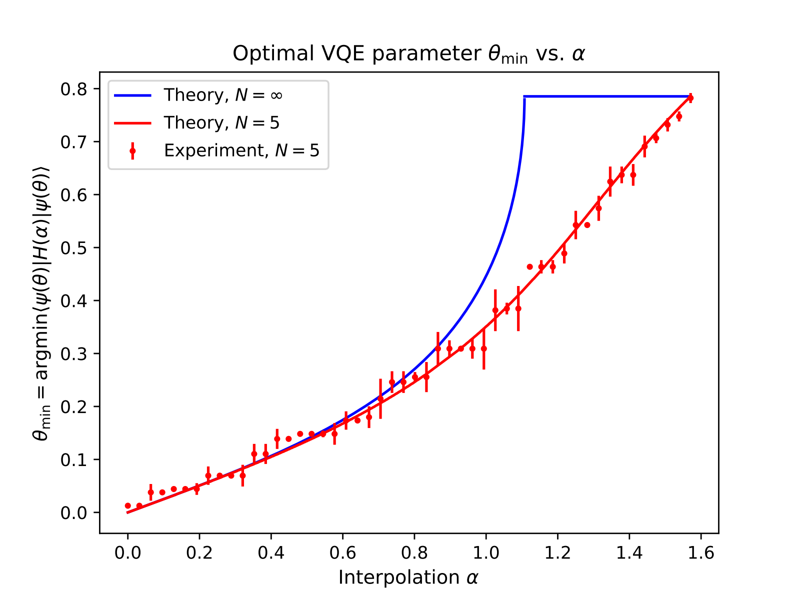
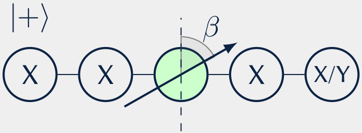
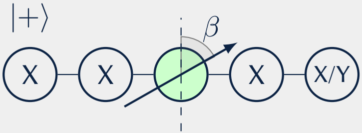




$$\sigma = \langle Z_3 \quad X_4 \quad Z_5 \rangle$$



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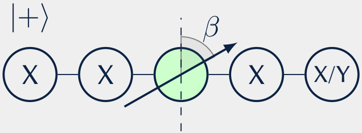
$$\text{(from } \mathcal{V}\text{): } \langle \overline{X} \rangle_+ = \cos(\beta),\, \langle \overline{Y} \rangle_+ = \nu \sin(\beta) \implies \frac{\langle \overline{Y} \rangle_+}{\langle \overline{X} \rangle_+} = \nu \tan(\beta)$$



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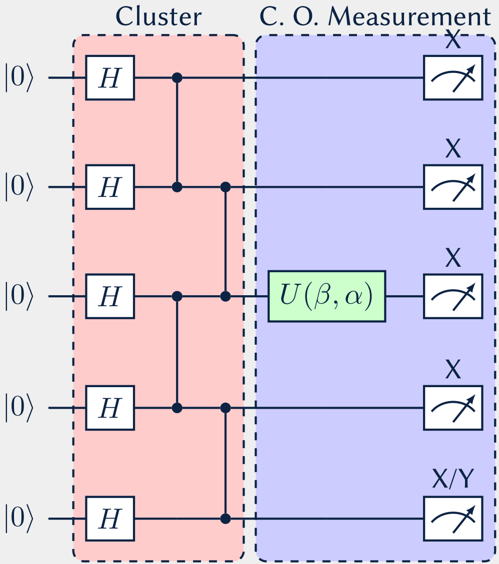
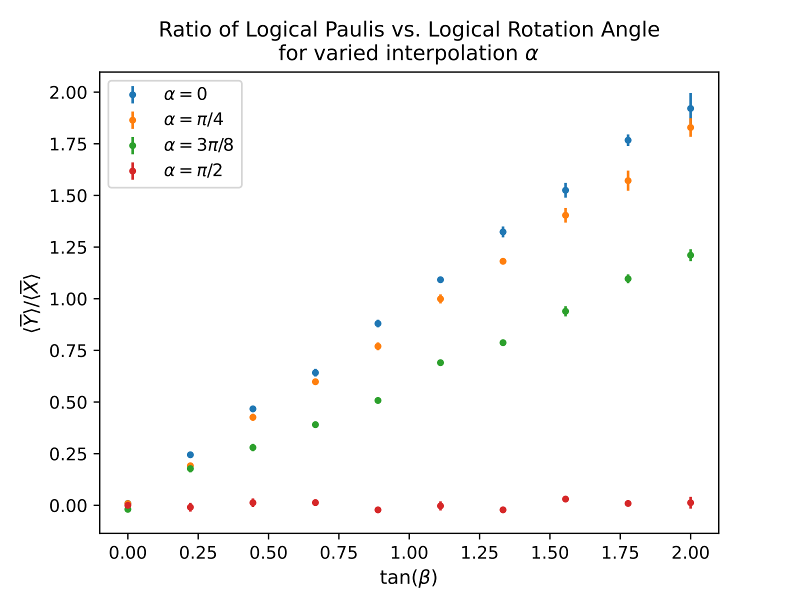
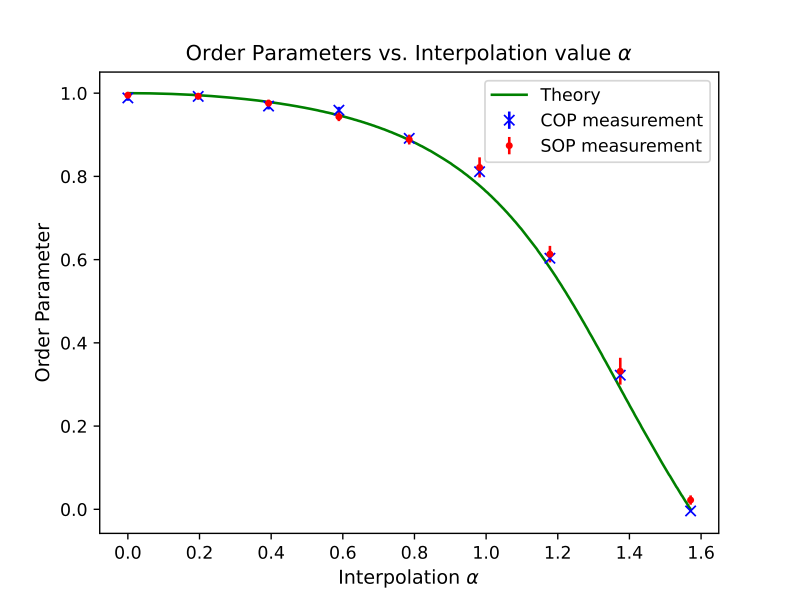
ν from MBQC, σ (for free) from VQE!

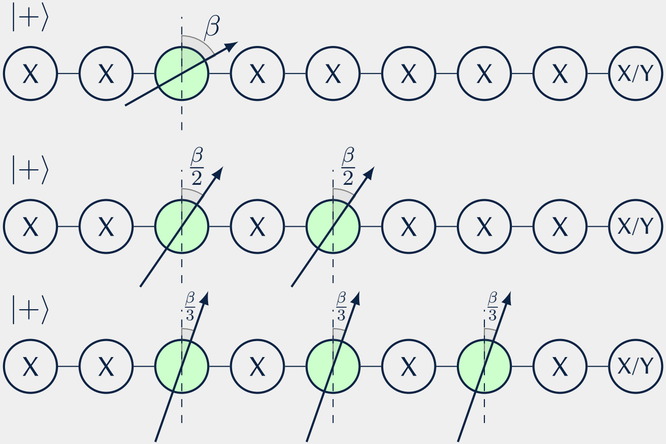
Experiment 1 - The Circuit Picture



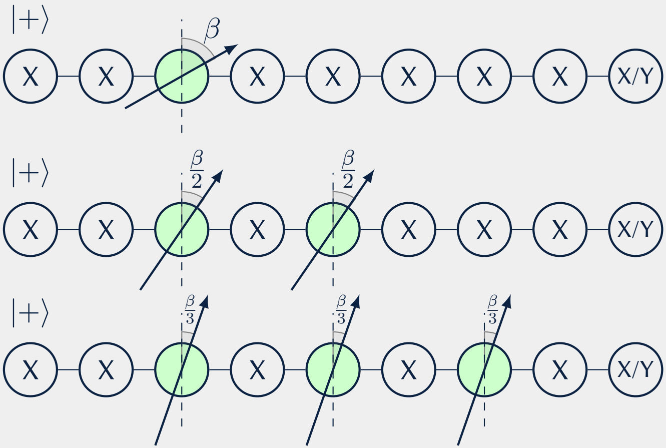

Ratio of Logical Paulis vs. Logical Rotation Angle
for varied interpolation α
$\alpha = 0$
$\alpha = \pi/4$
$\alpha = 3\pi/8$
$\alpha = \pi/2$
$\langle\overline{Y}\rangle/\langle\overline{X}\rangle$
$\tan(\beta)$
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00

Experiment 1 - Computational order = String order (Results)

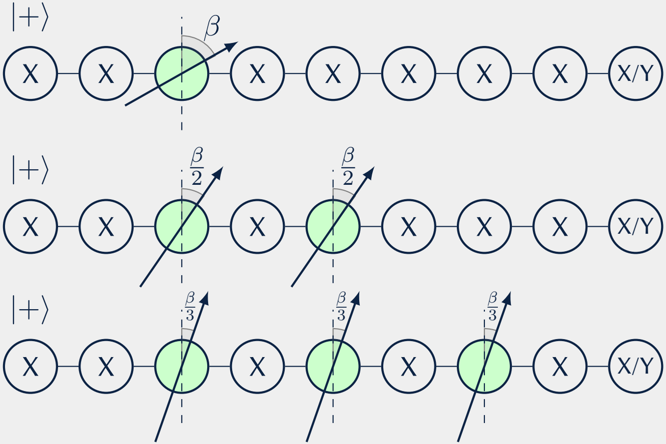



|+⟩
β
X
X
X
X
X
X
X
X/Y
|+⟩
β/2
β/2
X
X
X
X
X
X
X/Y
|+⟩
β/3
β/3
β/3
X
X
X
X
X
X/Y

Experiment 2 - How to measure divide and conquer



- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle\overline{X}(\beta)\rangle^2 - \langle\overline{Y}(\beta)\rangle^2$ in the three cases.



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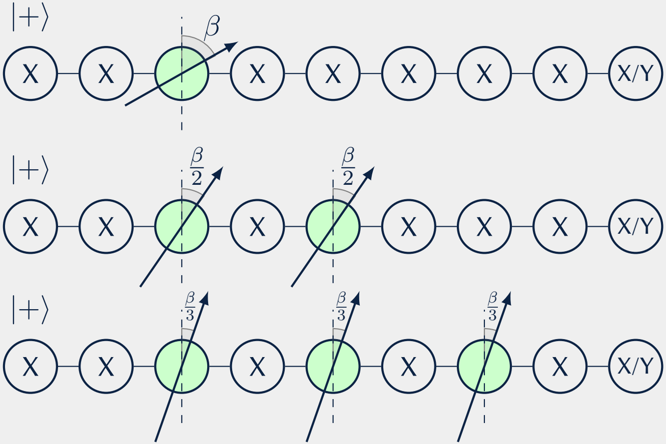
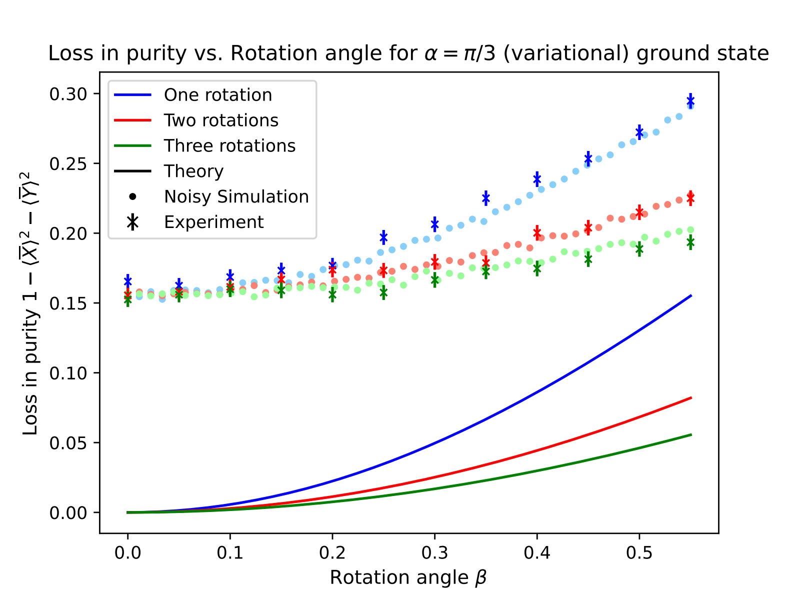
- For small angles β, verify LOP $\sim \frac{1}{m}$ (from $\mathcal{V}$).

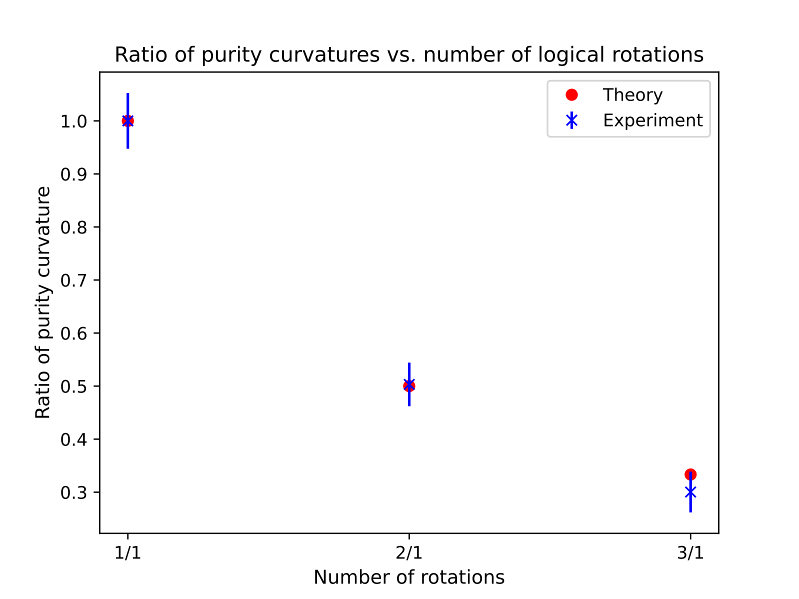
Experiment 2 - How to measure divide and conquer



- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle\overline{X}(\beta)\rangle^2 - \langle\overline{Y}(\beta)\rangle^2$ in the three cases.
- For small angles β, verify $\mathrm{LOP} \sim \frac{1}{m}$ (from $\mathcal{V}$).
- Fun bonus: No postselection!


Loss in purity vs. Rotation angle for $\alpha = \pi/3$ (variational) ground state
One rotation
Two rotations
Three rotations
Theory
Noisy Simulation
Experiment
Loss in purity $1 - \langle \overline{X} \rangle^2 - \langle \overline{Y} \rangle^2$
Rotation angle β
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.1
0.2
0.3
0.4
0.5

Experiment 2 - Divide and conquer (Results)



Experiment 3 - Resource state

- VQE ansatz $|\psi(\theta)\rangle$ has no length scale!

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- For this experiment we consider the simplest state with length scale:

$$|\Omega(\phi)\rangle = RX_2(\phi)RX_3(\phi)RX_{N-2}(\phi)RX_{N-1}(\phi)\prod_{i=2}^{N-3} RXX_{i,i+2}(\phi)\,|\mathcal{C}\rangle$$

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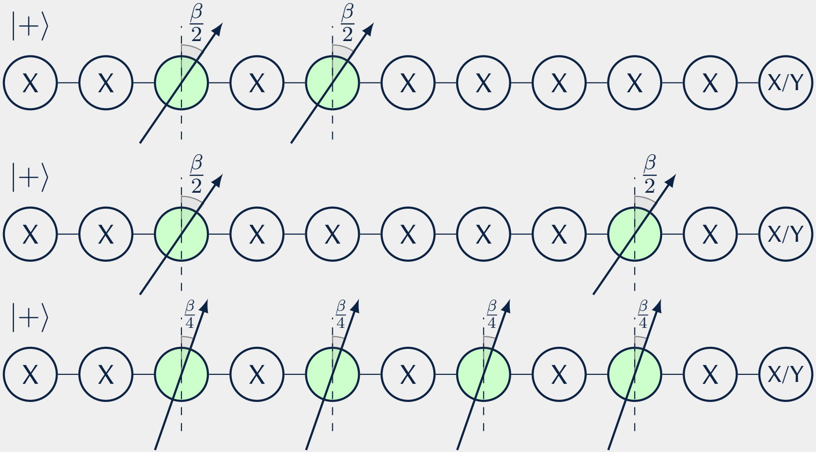
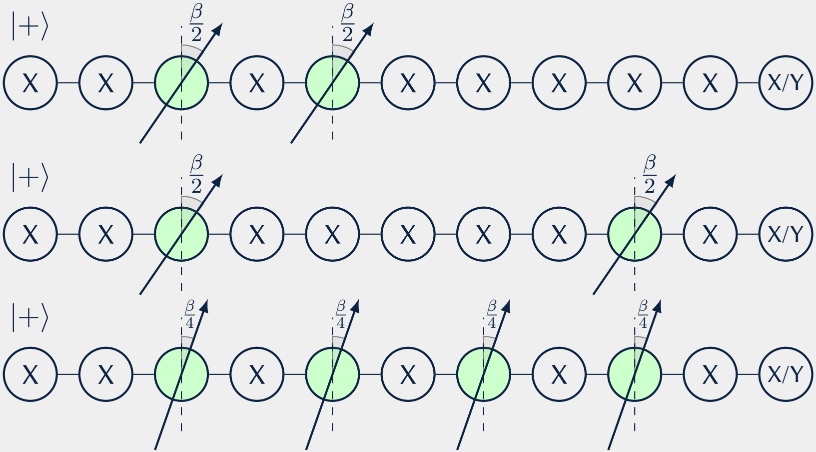
- String order parameters have convex decay, with:

$$\sigma(d) = \begin{cases} \cos^2(\phi) & d = 2 \\ \cos^4(\phi) & d > 2 \end{cases}$$

Experiment 3 - How to measure the counterintuitive regime





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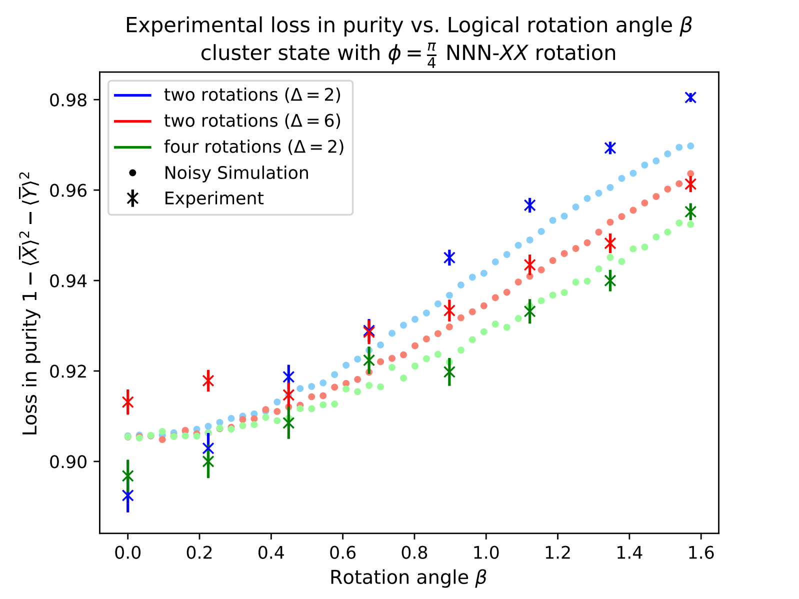
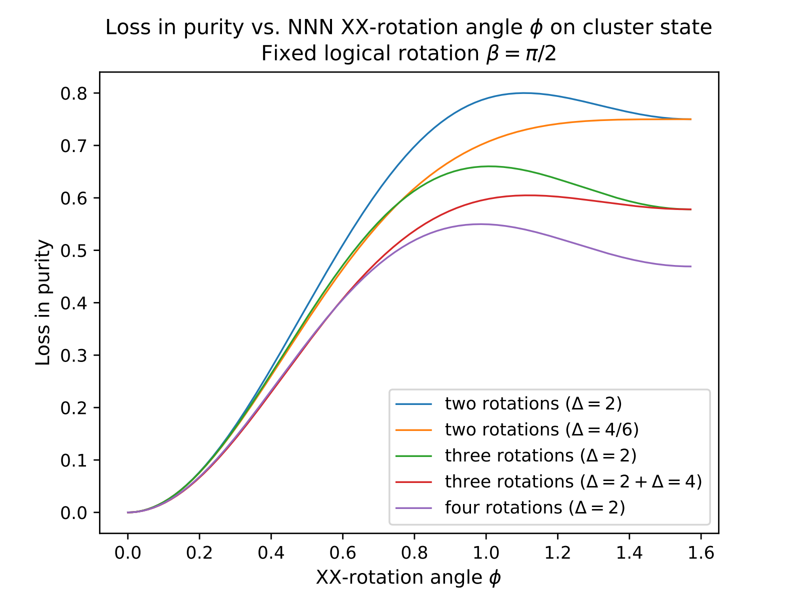
Experiment 3 - The counterintutive regime (Results)



Experiment 3 - The counterintutive regime (Theory)



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Thank you! Any questions?