Characterizing resource states and efficient regimes of MBQC on NISQ devices

ITP-Hannover Mini Workshop

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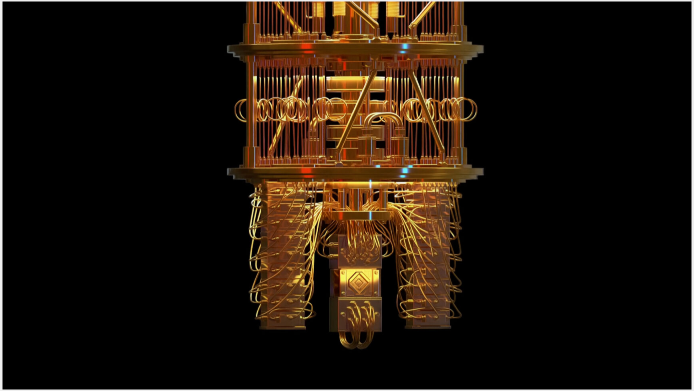
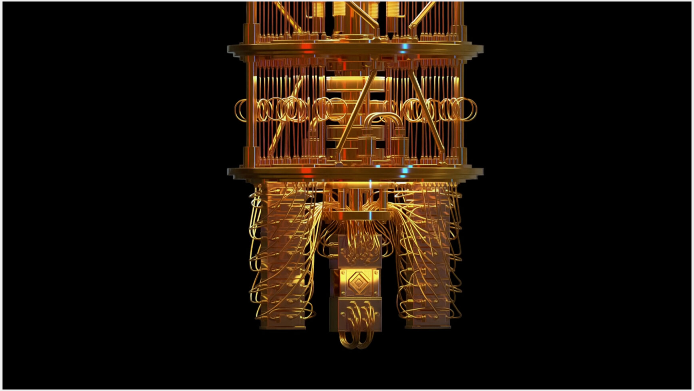
Motivating Questions



1. How do we characterize quantum computational speedup?
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

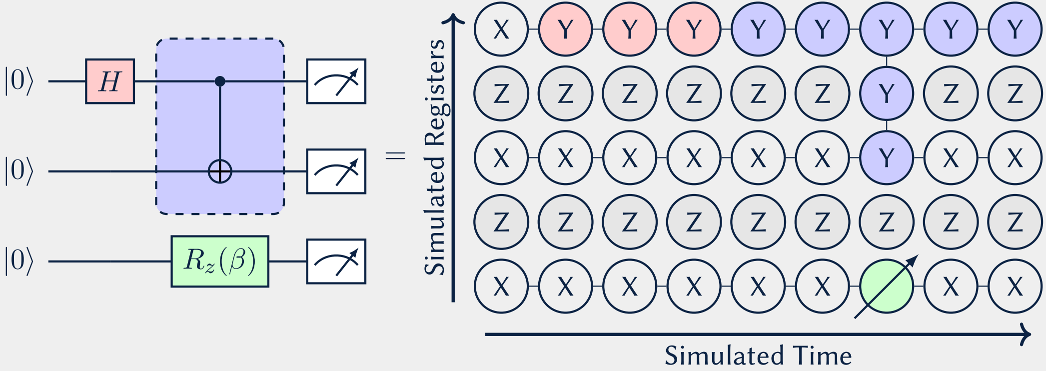
Image Credit: Quanta Magazine

Motivating Questions



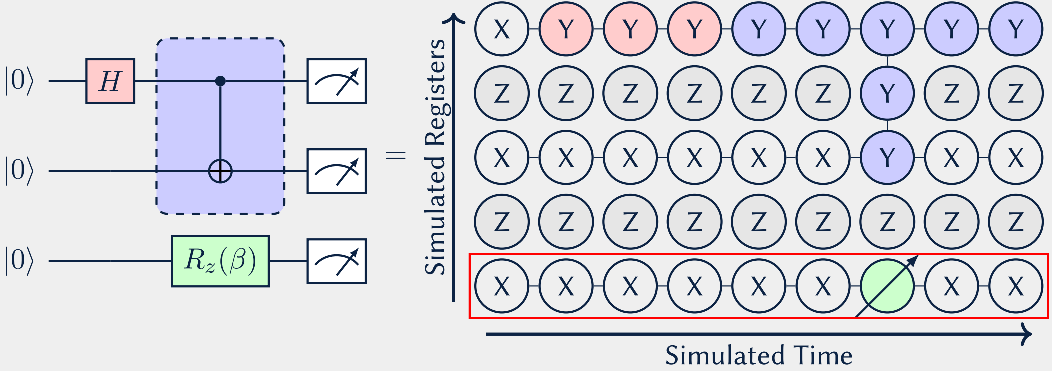
1. How do we characterize quantum computational speedup?
   - One route - Measurement-Based Quantum Computing
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Fun playground for physicists!

Image Credit: Quanta Magazine



	Gate Model	MBQC
Initialization	$\|00\ldots0\rangle$	(Universal) resource state
Evolution	Unitary Gates	(Adaptive) singe-qubit measurements

A One-Slide Review of MBQC



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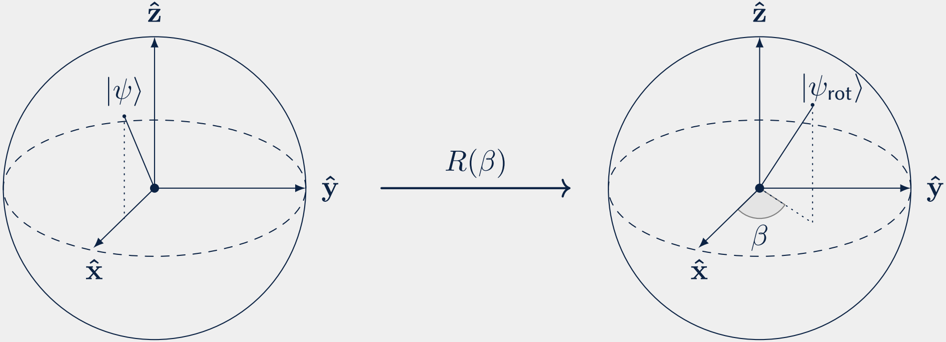
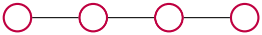


Ability to perform arbitrary single qubit unitaries (rotations) with high fidelity.

1D Resource States - 2 Examples and Interpolation

Universal Resource: Cluster State $|C\rangle$



Ground state of
$H_{\text{cluster}} = -\sum_i Z_{i-1} X_i Z_{i+1}$

Useless Resource: Product State $|+\rangle^{\otimes N}$

Ground state of
$H_{\text{product}} = -\sum_i X_i$

1D Resource States - 2 Examples and Interpolation

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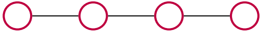

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Useless Resource: Product State $|+\rangle^{\otimes N}$

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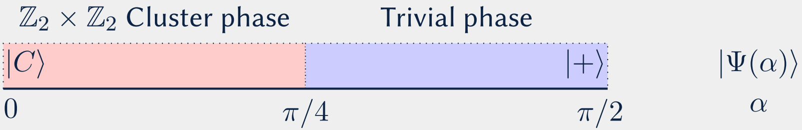
$H_{\text{product}} = -\sum_i X_i$

Computational order ground states $|\Psi(\alpha)\rangle$ of:

$$H(\alpha) = -\cos(\alpha) \sum_i Z_{i-1} X_i Z_{i+1} - \sin(\alpha) \sum_i X_i$$

Answer (for infinite systems):



- Computational power is uniform in symmetry-protected topological (SPT) phases.

1D Resource States - SPT Phases & Decoherence

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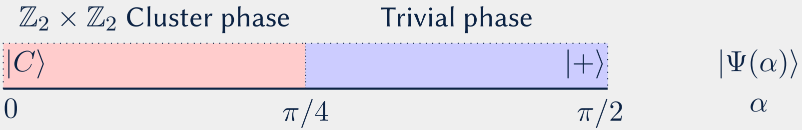

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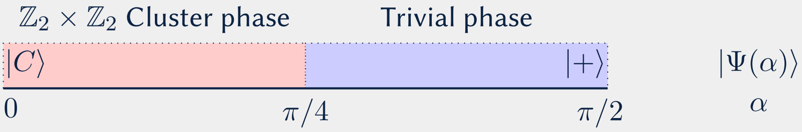

- Computational power is uniform in symmetry-protected topological (SPT) phases.
- The catch: Decoherence away from cluster state.
- Recently: New formalism for analyzing power in *finite* resource states.

- Desired logical rotation $\exp\left(-i\frac{\beta}{2}P\right)$ becomes a probabilistic channel:

$$\mathcal{V} = \frac{1+\nu}{2}\exp\left(-i\frac{\beta}{2}P\right) + \frac{1-\nu}{2}\exp\left(i\frac{\beta}{2}P\right)$$

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- ν is the computational order parameter, equivalent to σ the *string order parameter.*

1D Resource States - String Order & Decoherence

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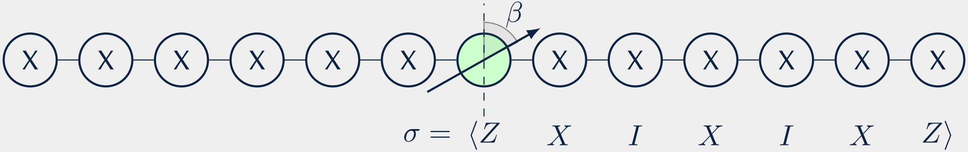
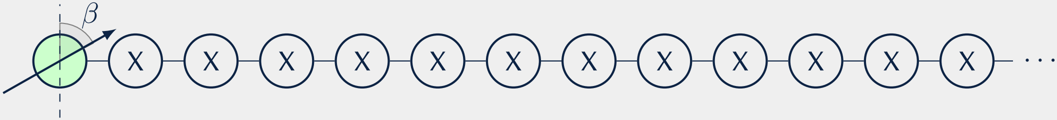
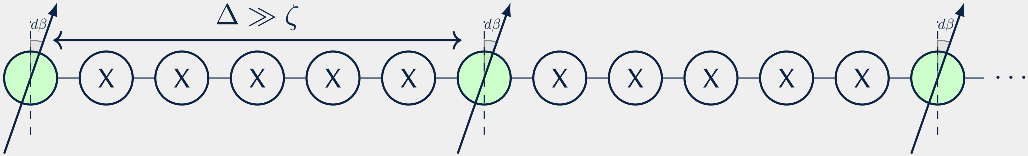


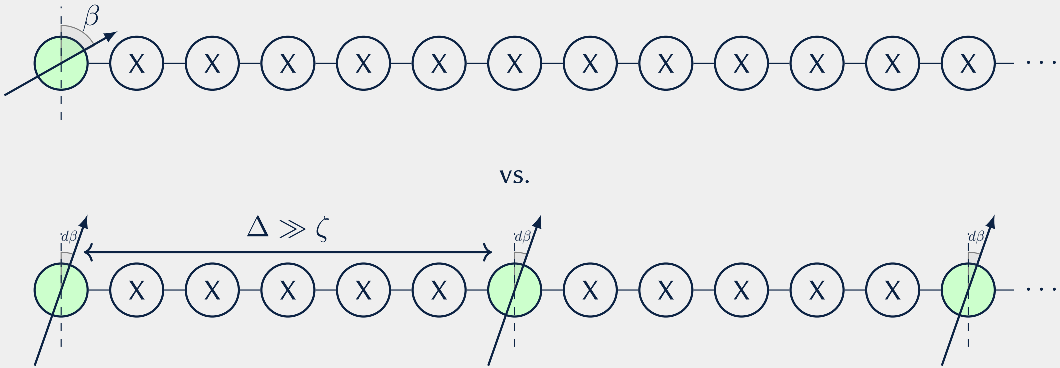


vs.

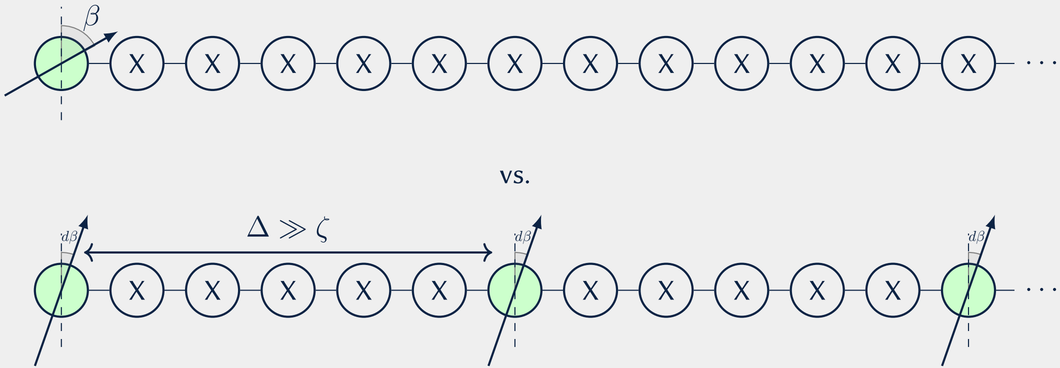


- Error is $O(\beta^2)$ - Dividing the rotation reduces error!

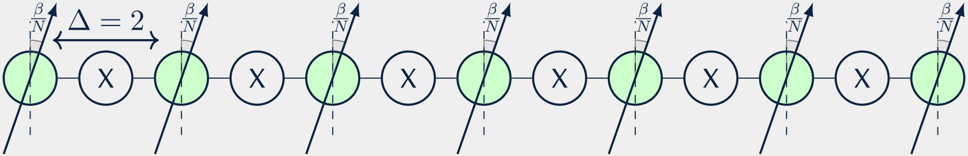
1D Resource States - Decoherence Management I



- Error is $O(\beta^2)$ - Dividing the rotation reduces error!
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as needed $\Rightarrow$ computational phases.



- Error is $O(\beta^2)$ - Dividing the rotation reduces error!
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as needed $\Rightarrow$ computational phases.
- Finite case: Tradeoff of rotation splitting and independence.



Optimal strategy: Split as much as possible ($\Delta = 2$), even if "counterintuitive".

Proposed Experiments

1. Computational order = String order
2. Decoherence management I - Divide and conquer
3. Decoherence management II - The counterintuitive regime

Recall $H(\alpha)$:

$$H(\alpha) = -\cos(\alpha)\sum_i Z_{i-1}X_iZ_{i+1} - \sin(\alpha)\sum_i X_i$$

Experiment 0 - Ground State Ansatz

Recall $H(\alpha)$:

$$H(\alpha) = -\cos(\alpha)\sum_i Z_{i-1}X_iZ_{i+1} - \sin(\alpha)\sum_i X_i$$

We consider the following variational ansatz:

$$|\psi(\theta)\rangle = \bigotimes_{i=2}^{N-1} T_i(\theta)\,|\mathcal{C}\rangle = \bigotimes_{i=2}^{N-1}(\cos(\theta)I_i + \sin(\theta)X_i)\,|\mathcal{C}\rangle$$

Motivated by symmetry, perturbation theory, and efficiency.

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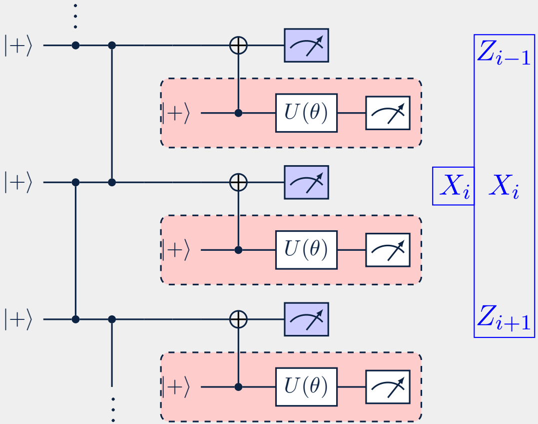
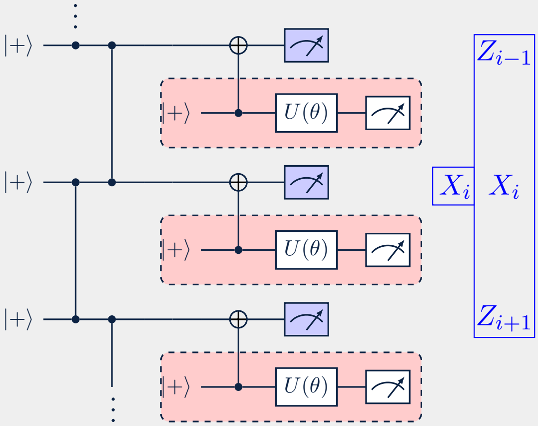
Motivated by symmetry, perturbation theory, and efficiency.
For a given value of α, find $|\psi(\theta)\rangle$ which minimizes:

$$\langle\psi(\theta)| H(\alpha) |\psi(\theta)\rangle = -\cos(\alpha) \sum_{i=1}^{N} \langle K_i = Z_{i-1} X_i Z_{i+1}\rangle_\theta - \sin(\alpha) \sum_{i=1}^{N} \langle X_i \rangle_\theta$$



Algorithm for finding $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$:

1. Prepare the cluster ring $|\mathcal{C}_N\rangle$.
2. Probabilistically implement (non-unitary) $T_i(\theta) = \cos(\theta) I_i + \sin(\theta) X_i$ on each site.
3. Measure X_i or $K_i = Z_{i-1} X_i Z_{i+1}$ on the prepared state to obtain $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$.



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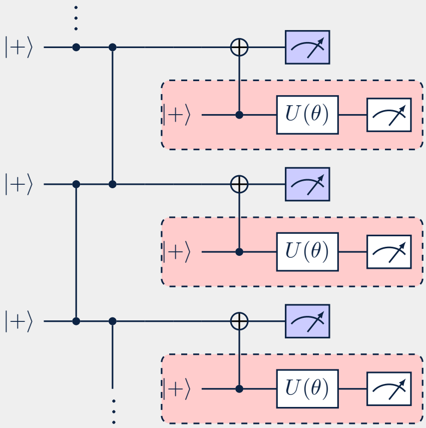
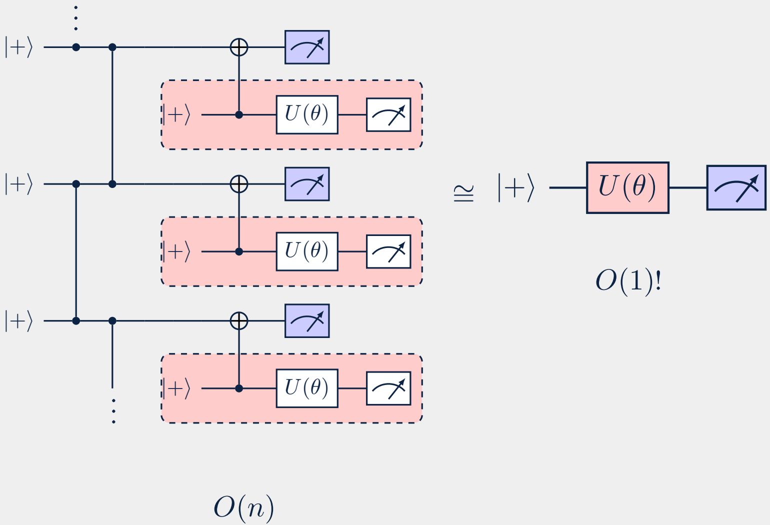
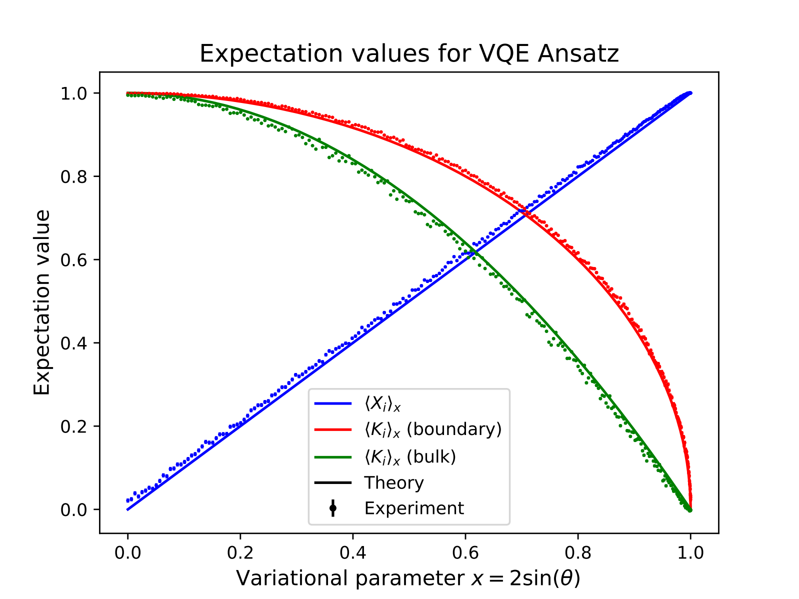
Then, various tricks with symmetry, half-teleportation, translation invariance...

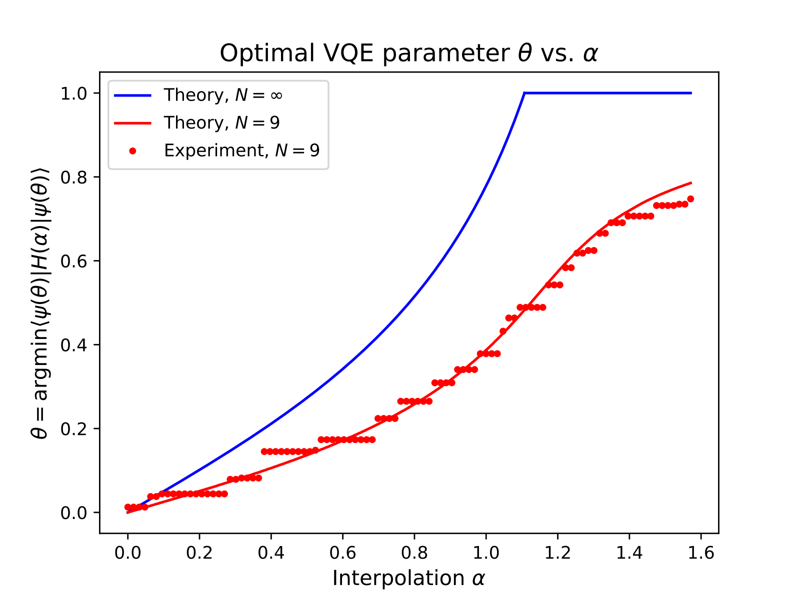

$|+\rangle$
$|+\rangle$
$U(\theta)$
$|+\rangle$
$|+\rangle$
$U(\theta)$
$|+\rangle$
$|+\rangle$
$U(\theta)$

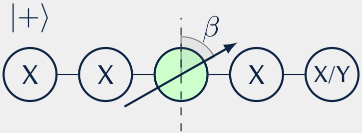
$O(n)$


$|+\rangle$
$|+\rangle$
$|+\rangle$
$|+\rangle$
$|+\rangle$
$|+\rangle$
$U(\theta)$
$U(\theta)$
$U(\theta)$
$\cong$
$|+\rangle$
$U(\theta)$
$O(1)!$
$O(n)$

Experiment 0 - State Preparation (Results)

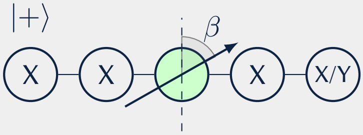



Optimal VQE parameter θ vs. α
Theory, $N = \infty$
Theory, $N = 9$
Experiment, $N = 9$
$\theta = \text{argmin}\langle\psi(\theta)|H(\alpha)|\psi(\theta)\rangle$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
Interpolation α



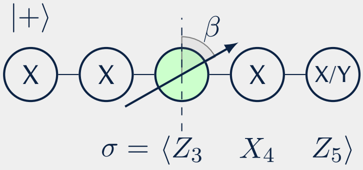
$$\sigma = \langle Z_3 \quad X_4 \quad Z_5 \rangle$$

Experiment 1 - How to measure computational and string order



$$\sigma = \langle Z_3 \quad X_4 \quad Z_5 \rangle$$

$$\text{(from } \mathcal{V}\text{)}: \langle \overline{X} \rangle_+ = \cos(\beta), \langle \overline{Y} \rangle_+ = \nu \sin(\beta) \implies \frac{\langle \overline{Y} \rangle_+}{\langle \overline{X} \rangle_+} = \nu \tan(\beta)$$



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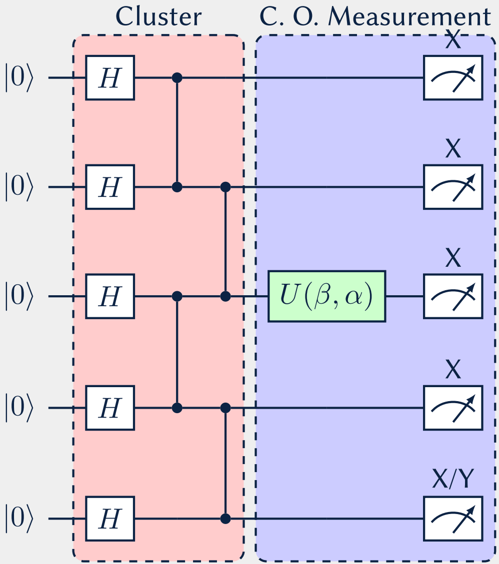
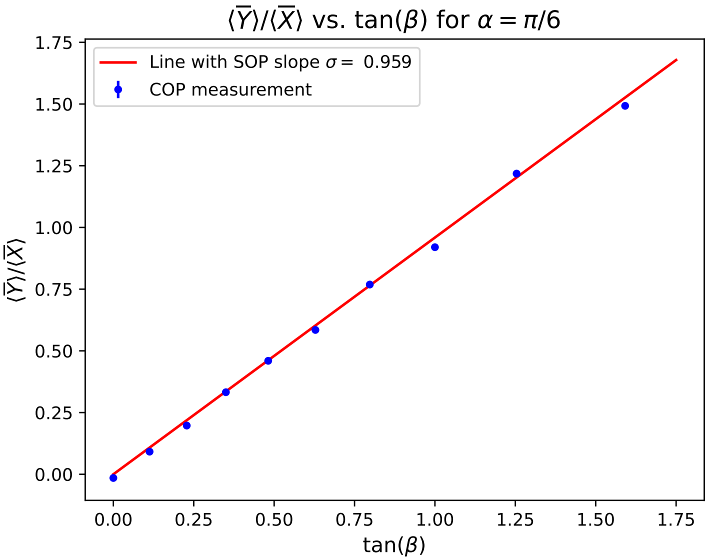
ν from MBQC, σ (for free) from VQE!


Cluster
C. O. Measurement
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
H
H
H
H
H
$U(\beta, \alpha)$
X
X
X
X
X/Y

Experiment 1 - Computational order = String order (Results)



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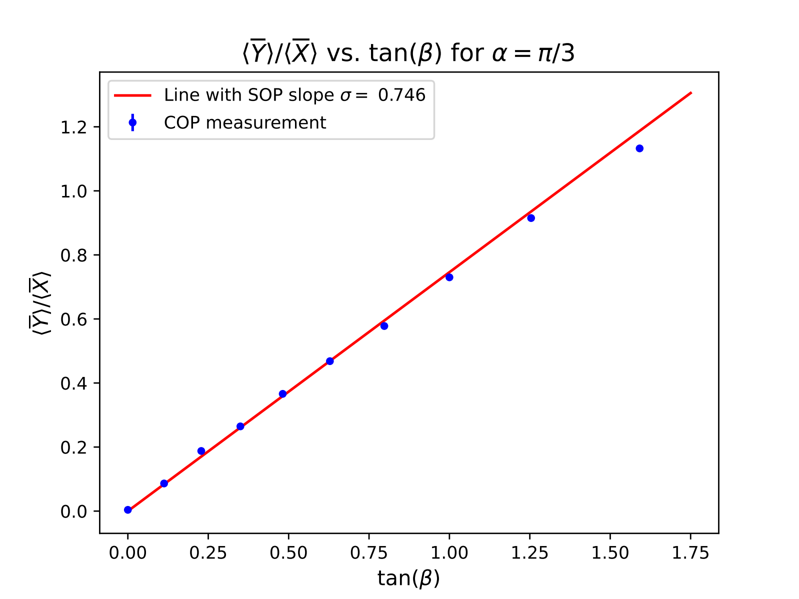
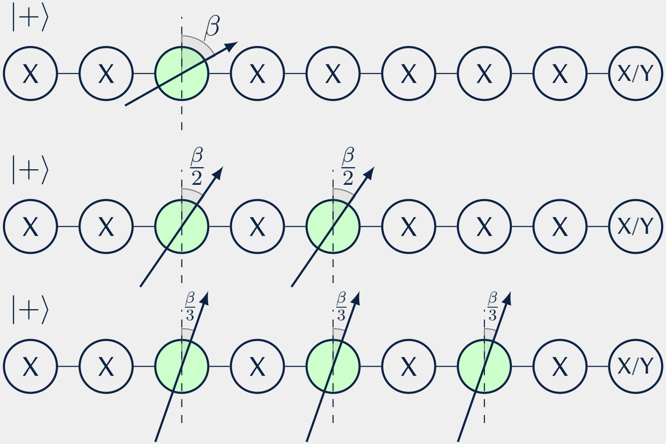
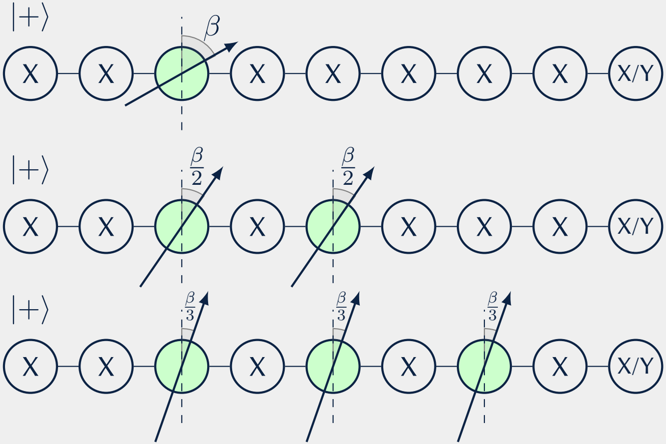
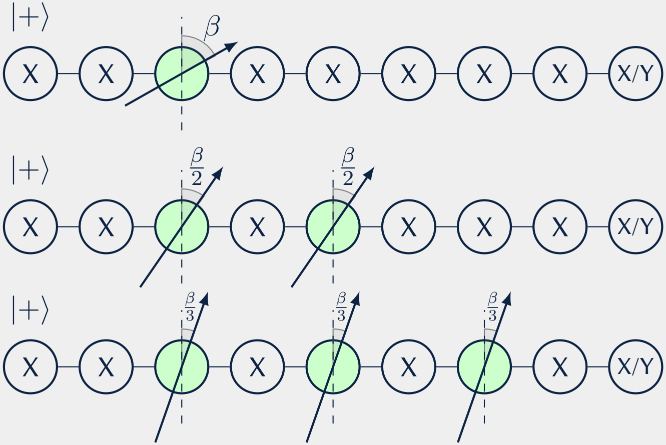


Experiment 2 - How to measure divide and conquer





- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle \overline{X}(\beta) \rangle^2 - \langle \overline{Y}(\beta) \rangle^2$ in the three cases.



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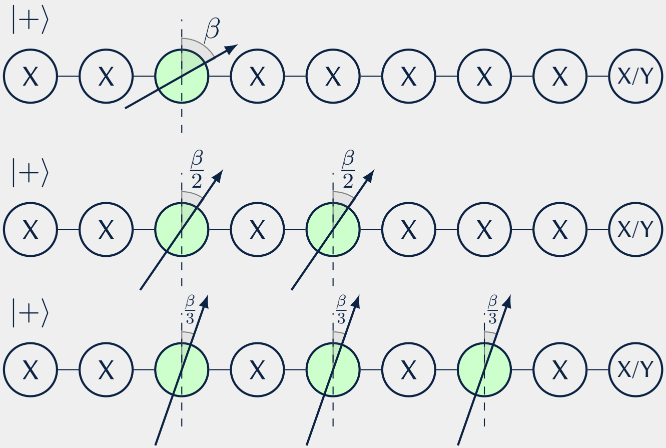
- For small angles β, verify LOP $\sim \frac{1}{N}$ (from $\mathcal{V}$).



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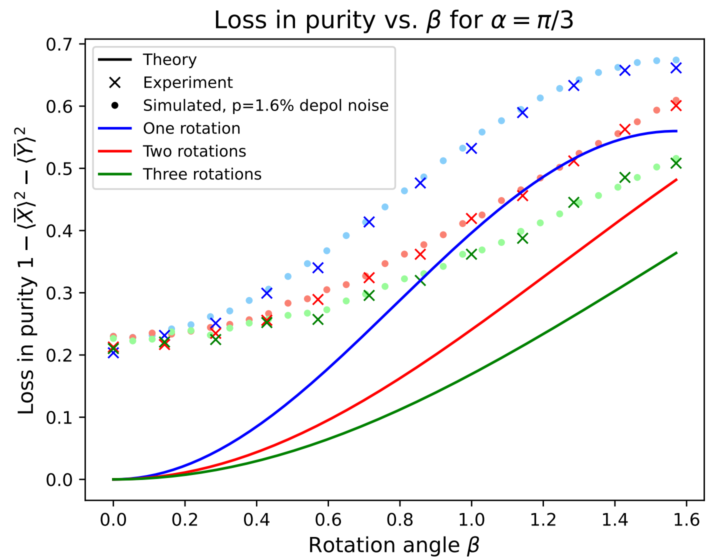
- For small angles β, verify LOP $\sim \frac{1}{N}$ (from $\mathcal{V}$).
- Fun bonus: No postselection!

Experiment 2 - Divide and conquer (Results)



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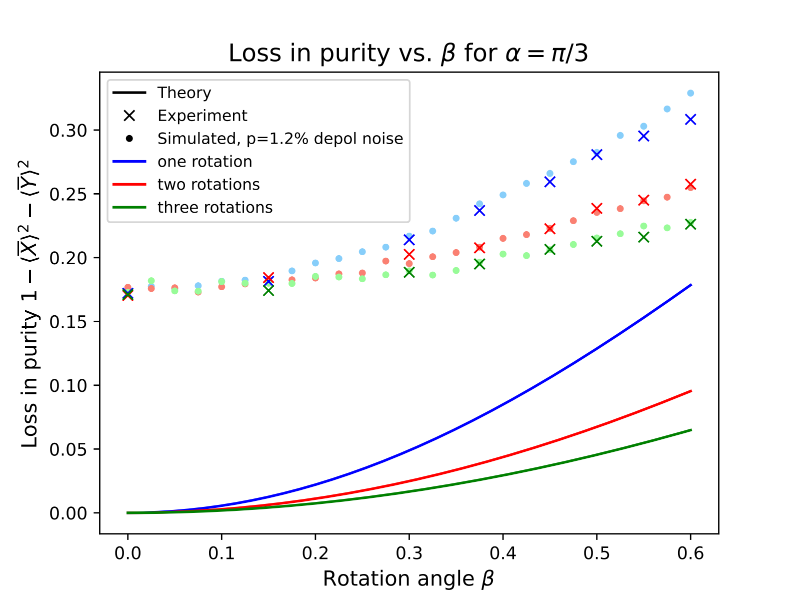

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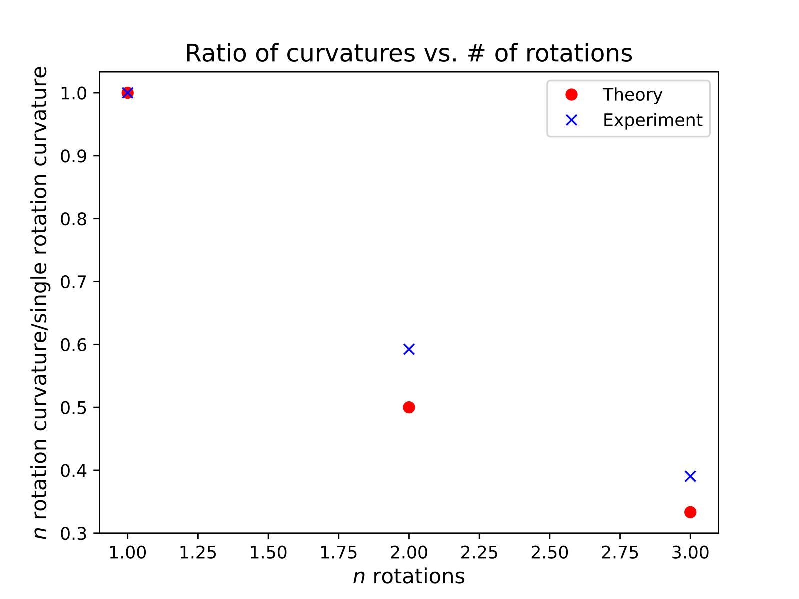
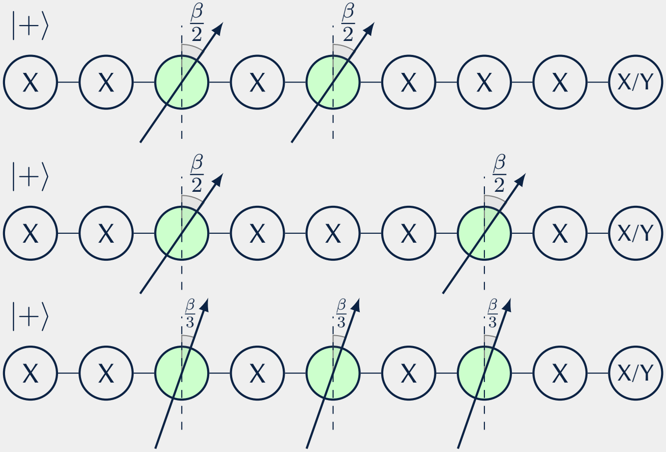


Experiment 3 - How to measure the counterintuitive regime (WIP)



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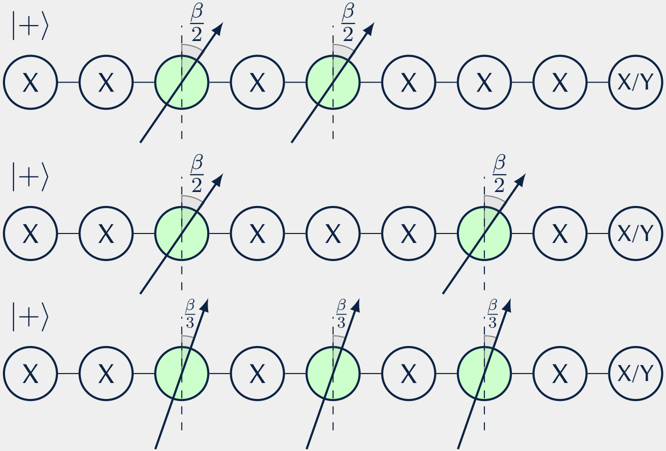

- Measure loss in purity $\mathrm{LOP}(\beta) = 1 - \langle \overline{X}(\beta) \rangle^2 - \langle \overline{Y}(\beta) \rangle^2$ in the three cases.
- Requires further (non-local) refinement to Ansatz - (e.g. $R_{X_k X_{k+2}}(\theta)$ gates)

Thank you!

(Insert group photo here)
Any questions?