A Simulation of a Simulation: Algorithms for Measurement-Based Quantum Computing Experiments

ASQC V

Rio Weil

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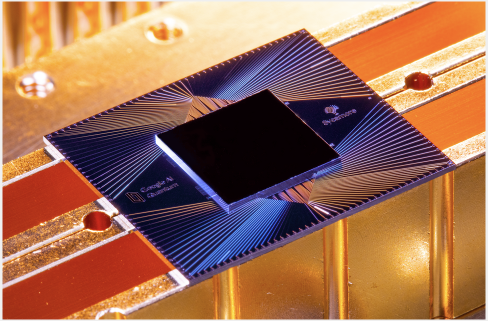
June 14, 2022

Outline

Motivating Questions

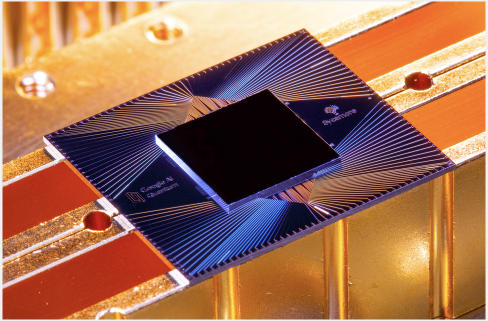


1. What is the source of quantum advantage?

2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

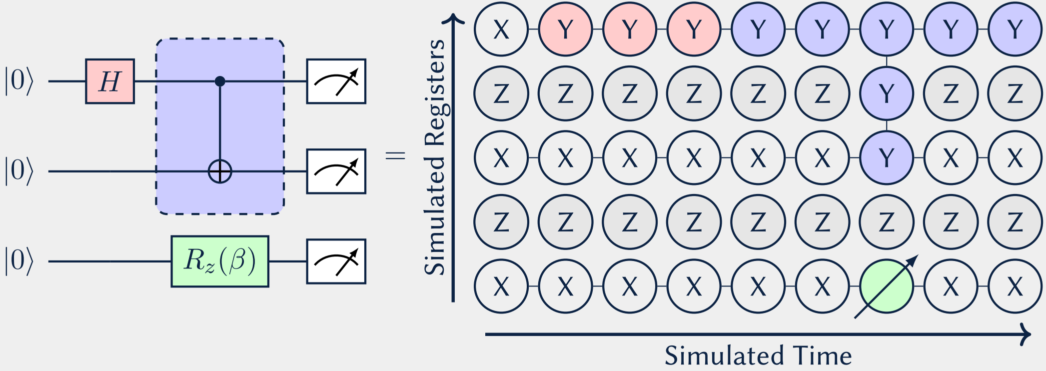
Image Credit: Erik Lucero/Google

Motivating Questions



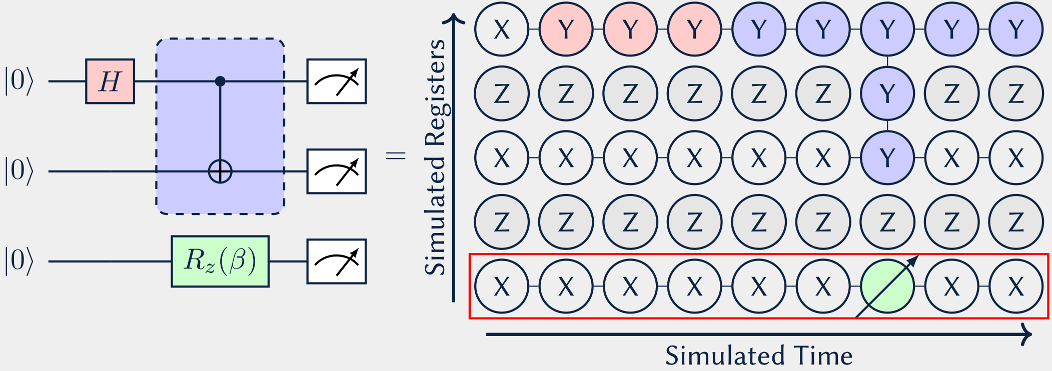
1. What is the source of quantum advantage?
   - Measurement-Based Quantum Computing - coming up!
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Active research area... and this project!

Image Credit: Erik Lucero/Google



	Gate Model	MBQC
Evolution Method	Unitary Gates	Single-Qubit measurements
“Power Source”	Intermediate Gates	Initial State

A One-Slide Review of MBQC



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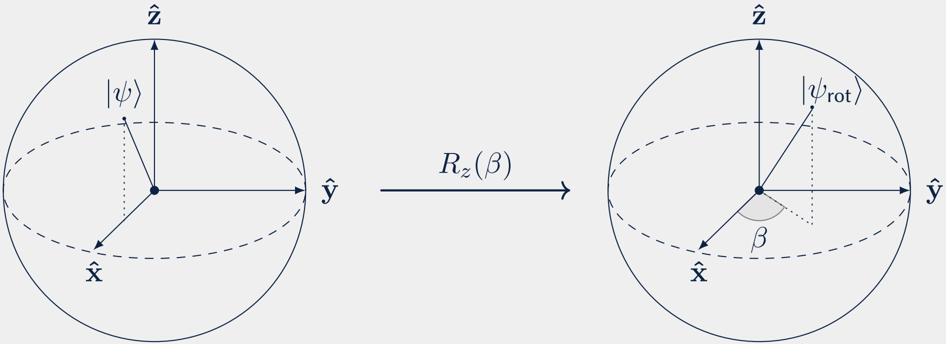
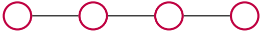


Ability to perform arbitrary single qubit unitaries - rotations.

Universal Resource: Cluster State $|C\rangle$



Ground state of
$H_{\text{cluster}} = -\sum_i Z_{i-1} X_i Z_{i+1}$

Useless Resource: Product State $|+\rangle^{\otimes N}$

Ground state of
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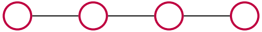

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Useless Resource: Product State $|+\rangle^{\otimes N}$

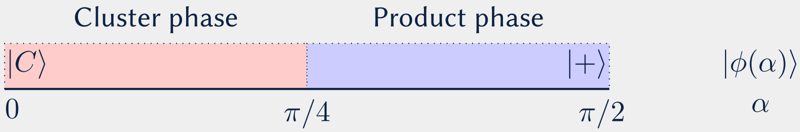
Ground state of
$H_{\text{product}} = -\sum_i X_i$

Question: Power of ground states $|\phi(\alpha)\rangle$ of:

$$H(\alpha) = -\cos(\alpha) \sum_i Z_{i-1} X_i Z_{i+1} - \sin(\alpha) \sum_i X_i ?$$

1D Resource States - Phase Diagram & Decoherence

Answer (for infinite systems):



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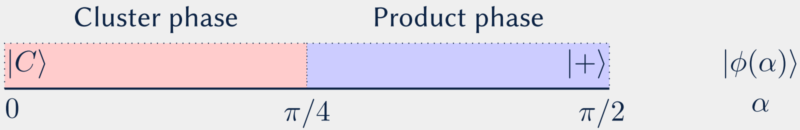

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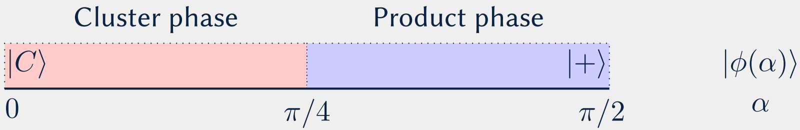
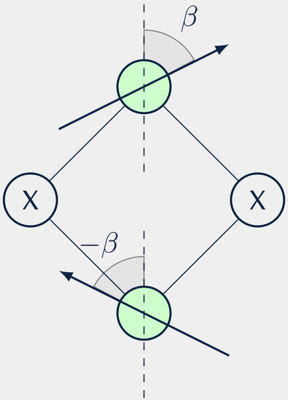


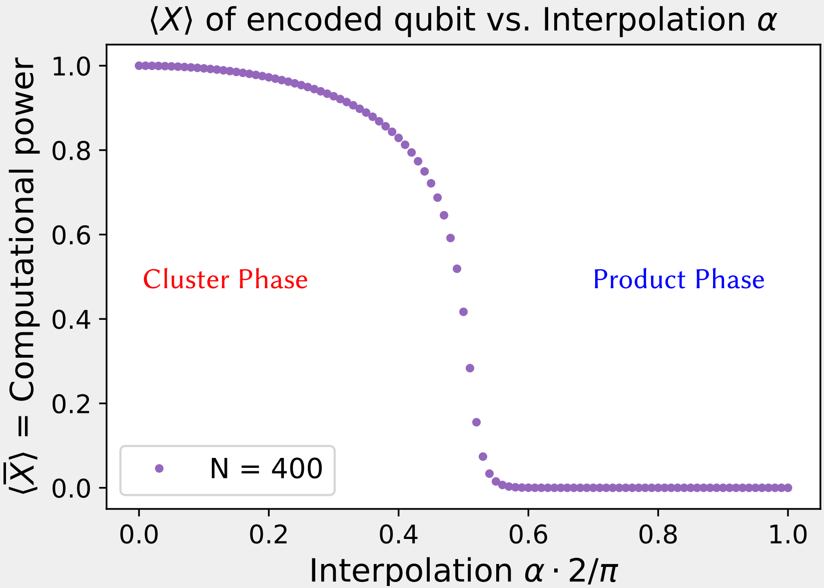
- Computational power is a property of (symmetry-protected topological) phases.
- The catch: Decoherence away from cluster state.
- Recently: New formalism for analyzing power in *finite* resource states.

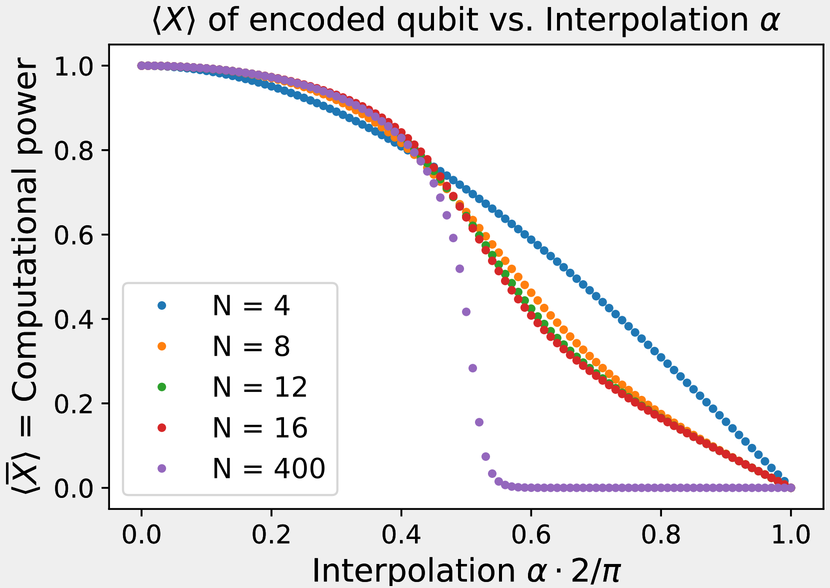


Demonstration: The rotation-counter rotation scheme.

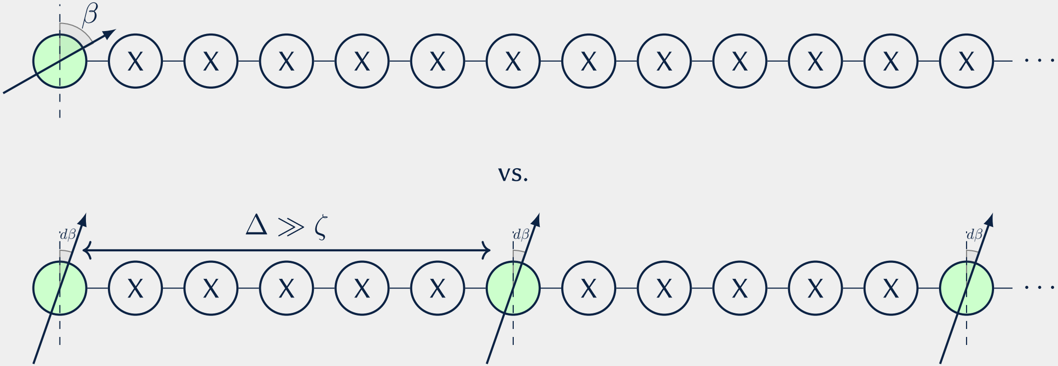
1. Prepare $|\phi(\alpha)\rangle$, and input $|+\rangle$.
2. Apply β rotation, and $-\beta$ counterrotation, separated by $\Delta = 2$.
3. Measure $\langle\overline{X}\rangle$: computational power.

1D Resource States - Predicted Results

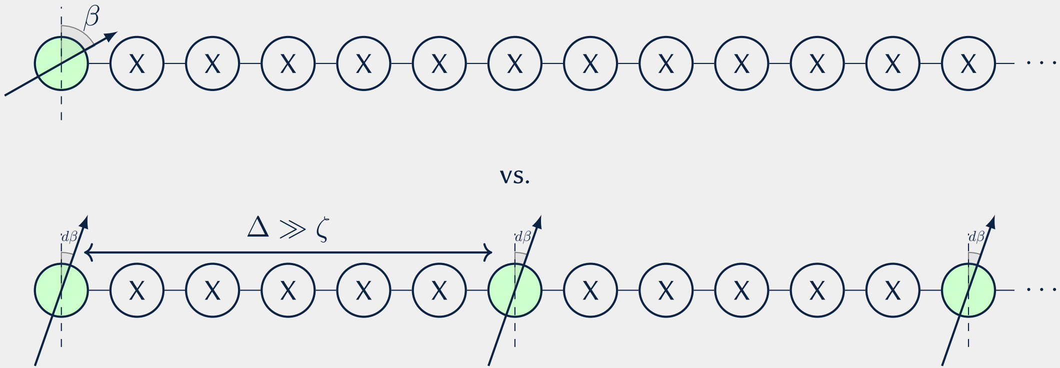




⟨X⟩ of encoded qubit vs. Interpolation α
⟨X̄⟩ = Computational power
Interpolation α · 2/π
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
N = 4
N = 8
N = 12
N = 16
N = 400



- Error is $O(\beta^2)$ - Dividing the rotation reduces error!



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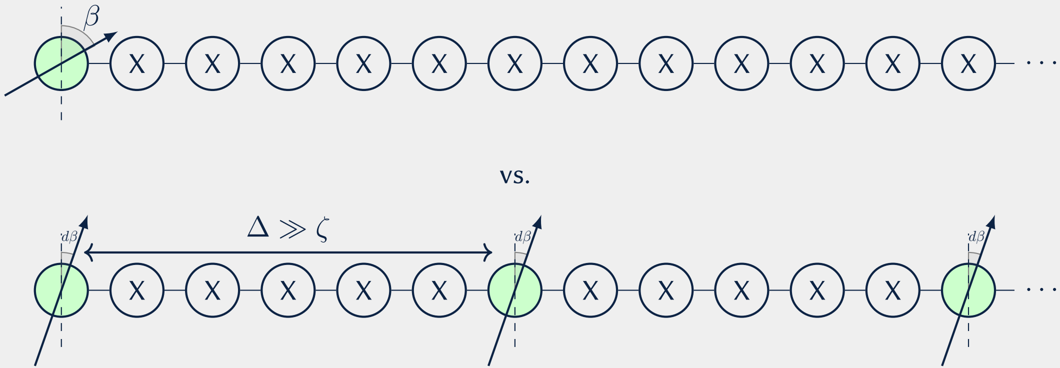
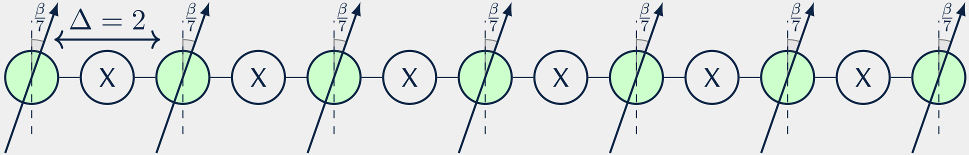
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as desired.

1D Resource States - Decoherence Management I



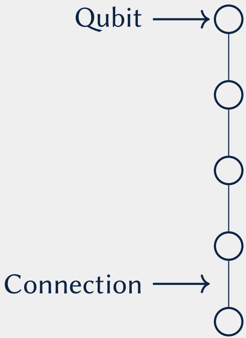
- Error is $O(\beta^2)$ - Dividing the rotation reduces error!
- Infinite case: Split as far apart ($\Delta \gg \zeta$) and as much as desired.
- Finite case: Tradeoff of rotation splitting and independence.

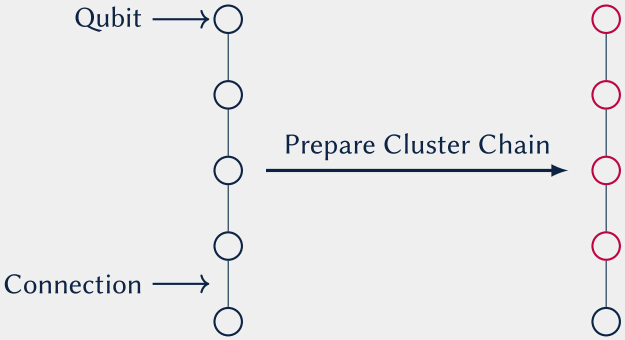


- Optimal strategy: Split as much as possible ($\Delta = 2$), even if "counterintuitive".

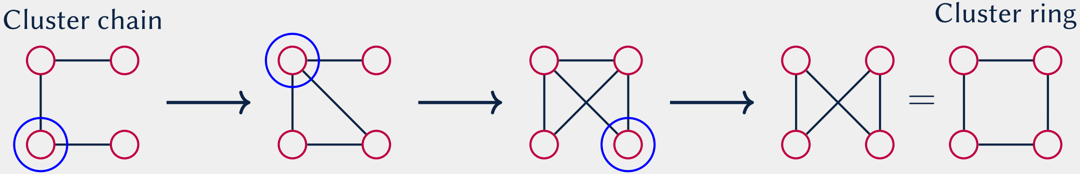
From Theory to Experiment - Desiderata

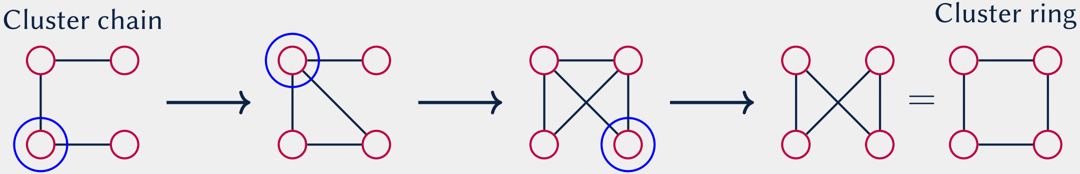
1. Computational power test - Rotation-counter rotation scheme (reproducing $\langle\overline{X}\rangle$ vs. α)
2. Decoherence management I - Divide and conquer
3. Decoherence management II - The counterintuitive regime


Qubit
Connection


Qubit
Connection
Prepare Cluster Chain

From Theory to Experiment - Local Complementation





Takeaway: Playing tricks to simulate a ring with a chain.

From Theory to Experiment - Producing Ground States

Recall $H(\alpha)$ and corresponding ground state $|\phi(\alpha)\rangle$:

$$H(\alpha) = -\cos(\alpha)\sum_i Z_{i-1}X_iZ_{i+1} - \sin(\alpha)\sum_i X_i$$

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Further, we can exchange unitarity for a simpler representation:

$$U(\alpha) \cong T(\alpha) = \text{only } I\text{s and } X\text{s}$$

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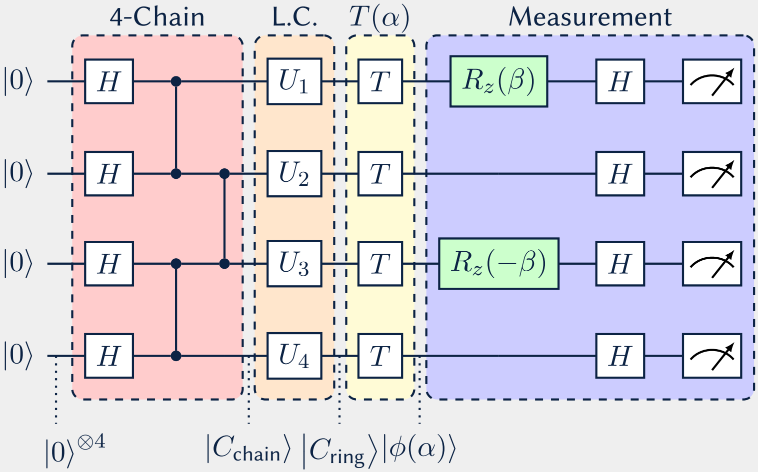
Further, we can exchange unitarity for a simpler representation:

$$U(\alpha) \cong T(\alpha) = \text{only } I\text{s and } X\text{s}$$

Finally, we can assume T is local, so:

$$T(\alpha) = \bigotimes_{i=1}^{N}(aI_i + bX_i)$$

With these simplifications, $T(\alpha)$ can be found via classical optimization, for small rings.


4-Chain
L.C.
$T(\alpha)$
Measurement
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
H
H
H
H
U_1
U_2
U_3
U_4
T
T
T
T
$R_z(\beta)$
$R_z(-\beta)$
H
H
H
H
$|0\rangle^{\otimes 4}$
$|C_{\text{chain}}\rangle$
$|C_{\text{ring}}\rangle$
$|\phi(\alpha)\rangle$

Problem: $T(\alpha)$ is non-unitary in general. But quantum gates are unitary.

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- Solution: Redraw some brackets:

$$p(\mathbf{j}) = |\langle \mathbf{j}|\phi(\alpha)\rangle|^2 = |\langle \mathbf{j}| [T(\alpha) |C\rangle]|^2 = |[\langle \mathbf{j}| T(\alpha)] |C\rangle|^2 = \left|\left\langle T^\dagger(\alpha)\mathbf{j}\middle|C\right\rangle\right|^2$$

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- Conclusion: Don't implement $T(\alpha)$ at all. Instead, measure $T^\dagger(\alpha)\,|\mathbf{j}\rangle$ on $|C\rangle$ instead!

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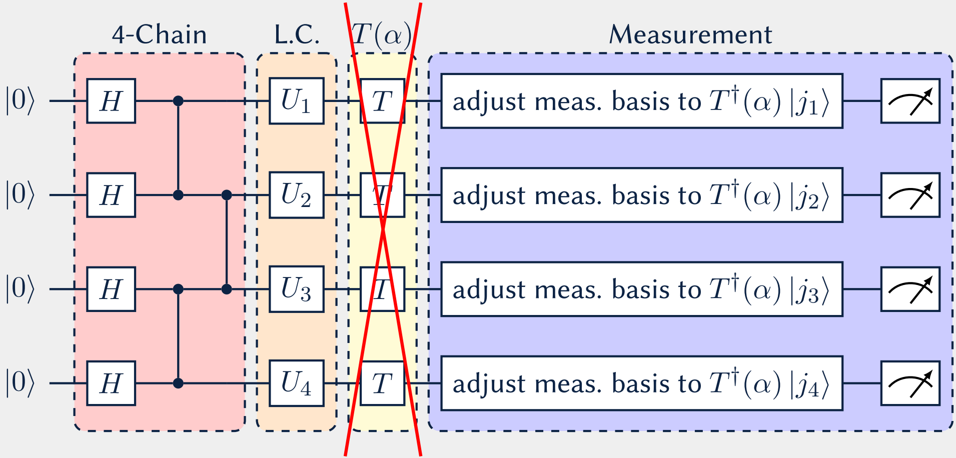
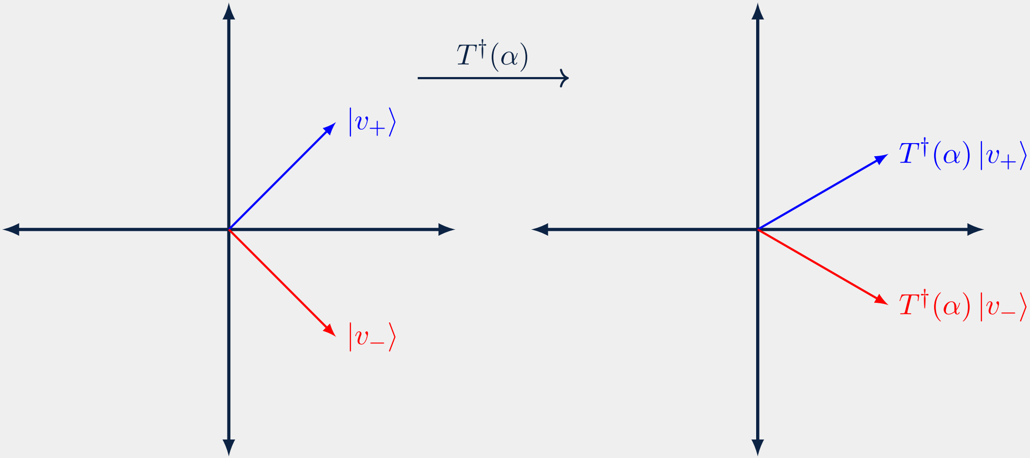
- Conclusion: Don't implement $T(\alpha)$ at all. Instead, measure $T^\dagger(\alpha) |\mathbf{j}\rangle$ on $|C\rangle$ instead!
- Takeaway: Problem Decomposition.

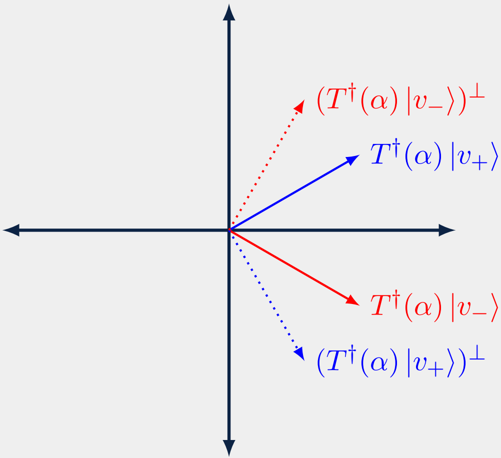
The Alluded Algorithm - Revised Goal

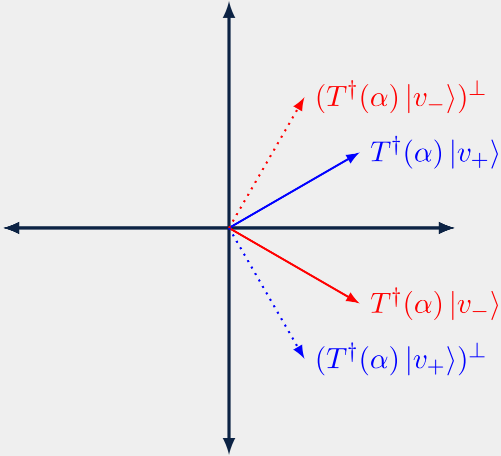


The Task: Develop algorithm to make circuits and post-process measurement outcomes, obtain results *as if* $T(\alpha)$ had been implemented.

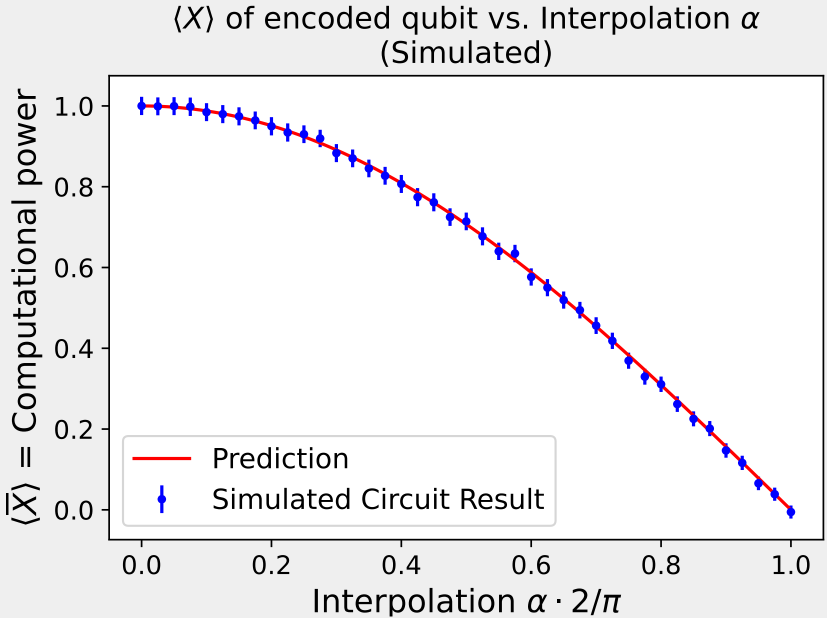


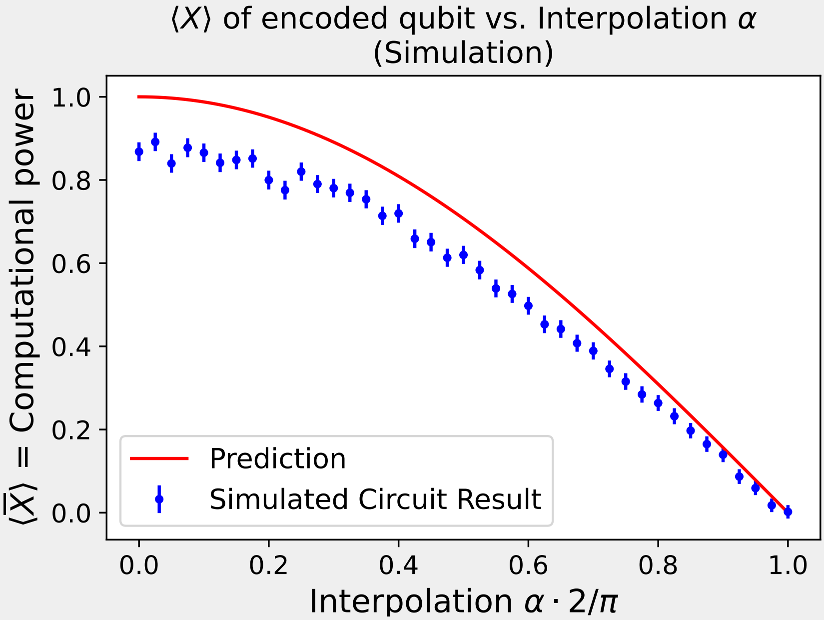
$|v_+\rangle$
$|v_-\rangle$
$T^\dagger(\alpha)$
$T^\dagger(\alpha)\,|v_+\rangle$
$T^\dagger(\alpha)\,|v_-\rangle$


$(T^\dagger(\alpha)\,|v_-\rangle)^\perp$
$T^\dagger(\alpha)\,|v_+\rangle$
$T^\dagger(\alpha)\,|v_-\rangle$
$(T^\dagger(\alpha)\,|v_+\rangle)^\perp$



Resolution: Sequence of multiple experiments, combining/processing the outcomes.


⟨X⟩ of encoded qubit vs. Interpolation α
(Simulated)
⟨X̄⟩ = Computational power
Interpolation α · 2/π
Prediction
Simulated Circuit Result
0.0
0.2
0.4
0.6
0.8
1.0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- Current setup: coefficients for $T(\alpha)$ found classically, and $T(\alpha)$ applied classically via post-processing.

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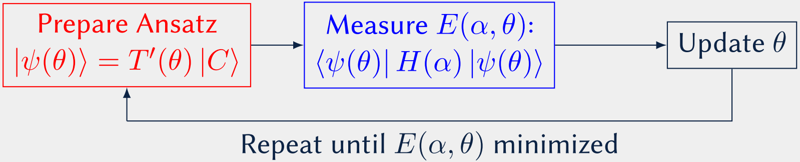
- More quantum mechanical (and more generalizable): Find $T(\alpha)$ on a quantum computer.

Making the Experiment "More Quantum" - Using VQE

- Current setup: coefficients for $T(\alpha)$ found classically, and $T(\alpha)$ applied classically via post-processing.
- More quantum mechanical (and more generalizable): Find $T(\alpha)$ on a quantum computer.
- Method: Variational Quantum Eigensolver



- Minimization (for a given α) yields θ for which $T'(\theta) = T(\alpha)$.

Making the Experiment "More Quantum" - VQE Ansatz

- Recall the target form of $T(\alpha)$ which satisfies $|\phi(\alpha)\rangle = T(\alpha)\,|C\rangle$:

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- Natural Ansatz:

$$|\psi(\theta)\rangle = T'(\theta)\,|C\rangle = \bigotimes_{i=1}^{4} T_i'(\theta)\,|C\rangle = \left(\bigotimes_{i=1}^{4} \cos(\theta)I_i + \sin(\theta)X_i\right)|C\rangle$$

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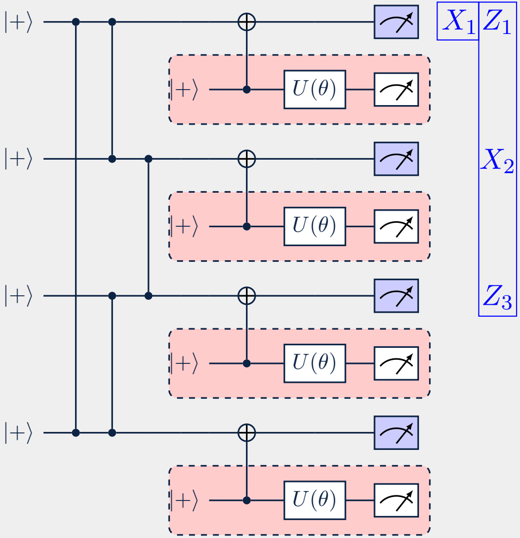
- Per-site energy to minimize:

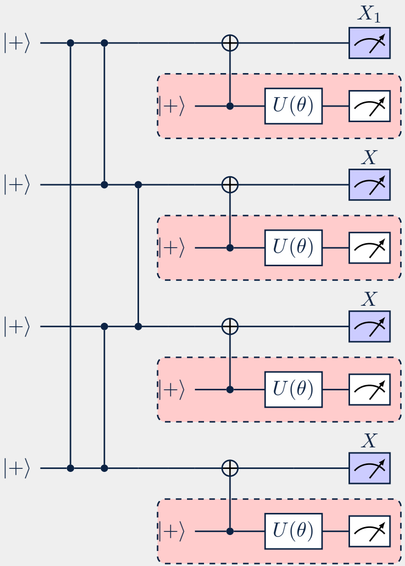
$$\langle E_i \rangle_\theta = -\cos(\alpha)\,\langle K_i = Z_{i-1} X_i Z_{i+1} \rangle_\theta - \sin(\alpha)\,\langle X_i \rangle_\theta$$



Algorithm for finding $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$:

1. Prepare the cluster ring $|C_4\rangle$.
2. Probabilistically implement (non-unitary) $T_i'(\theta) = \cos(\theta) I_i + \sin(\theta) X_i$ on each site.
3. Measure X_1 or $K_2 = Z_1 X_2 Z_3$ on the prepared state to obtain $\langle X_i \rangle_\theta$ / $\langle K_i \rangle_\theta$.

Making the Experiment "More Quantum" - Cascading Simplification



Large + Inefficient

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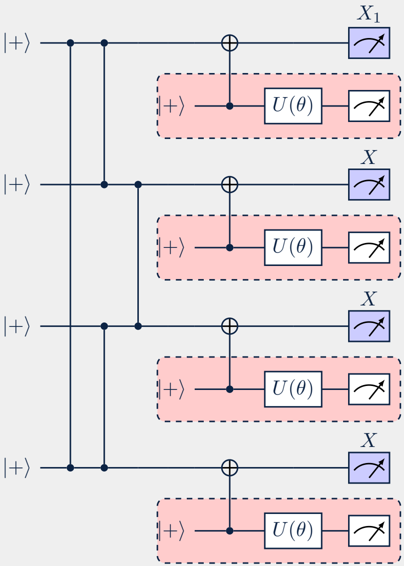

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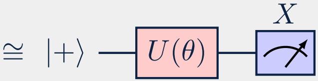
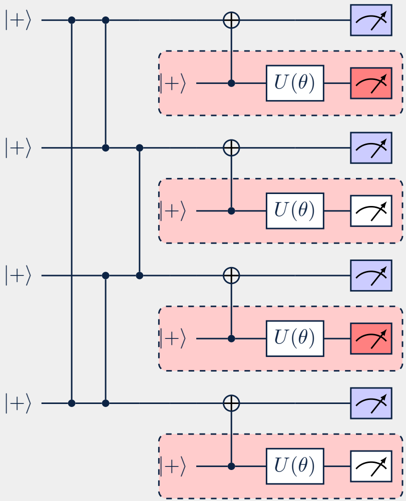


Small + Simple!

Making the Experiment "More Quantum" - Simplification Idea I



"Erroneous" outcome

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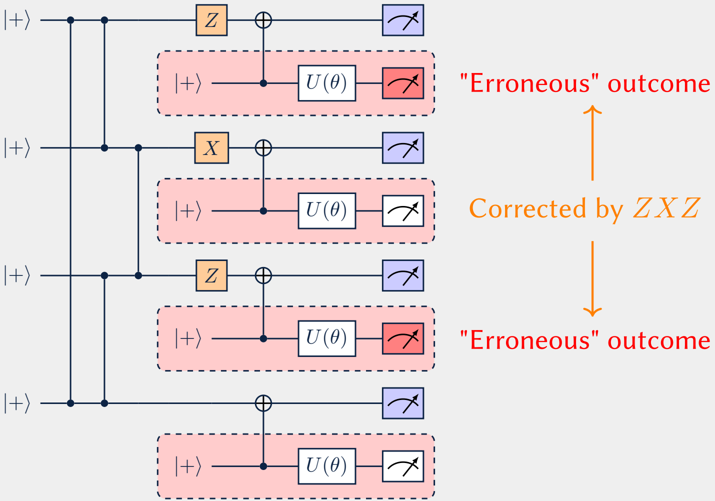
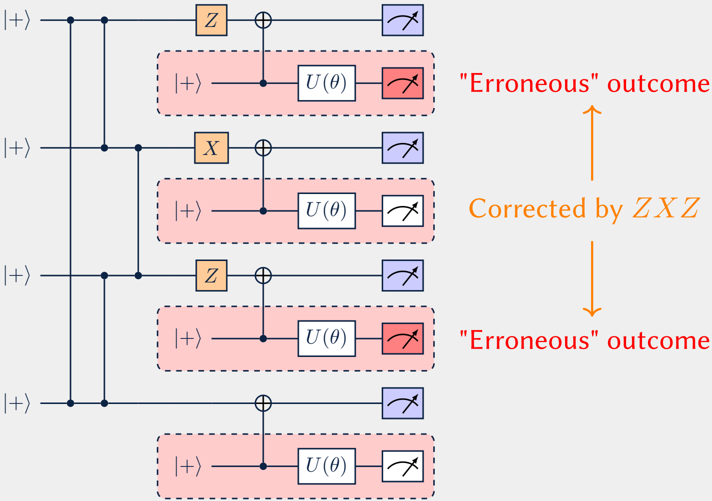
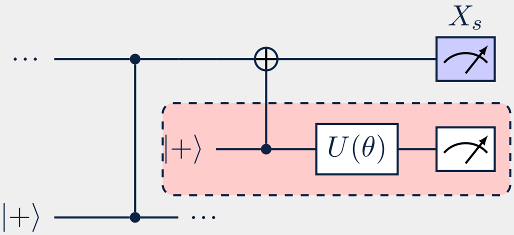


- (Some) errors can be corrected by pushing through cluster stabilizers ZXZ.



- (Some) errors can be corrected by pushing through cluster stabilizers ZXZ.
- Gain: Error probability doesn't increase exponentially with ring size, is instead constant.

Simplification of “Unit cell” of VQE Circuit:



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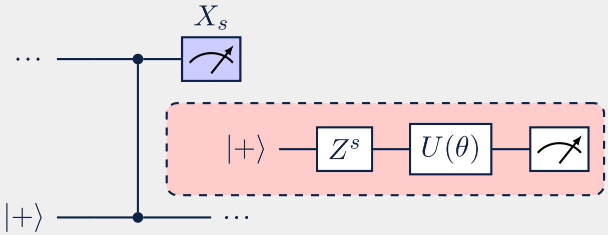

- Qubits disentangle...

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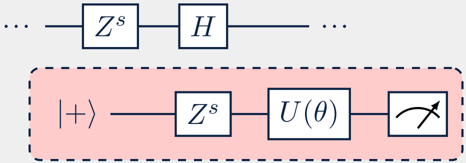

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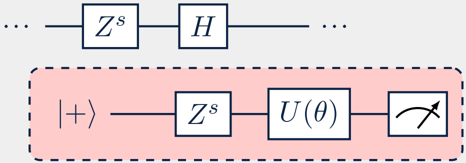
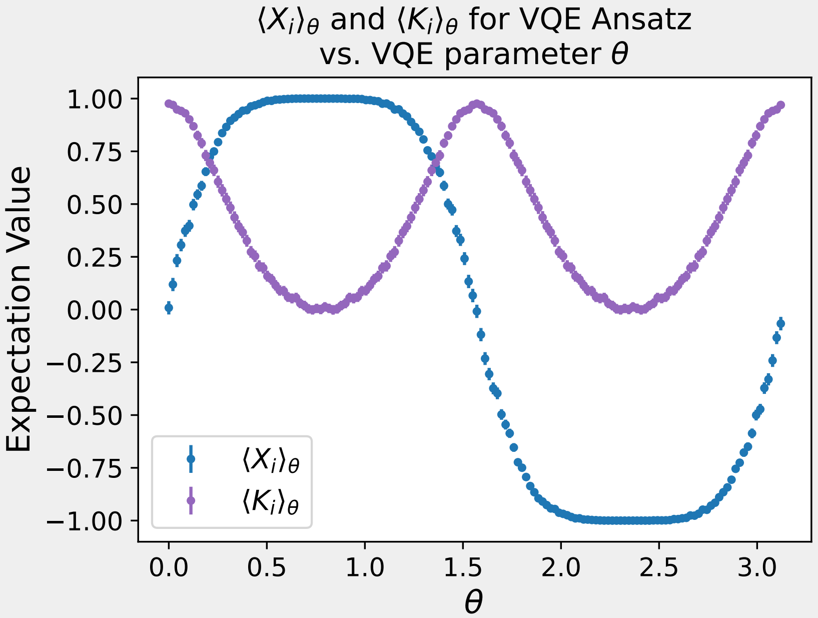
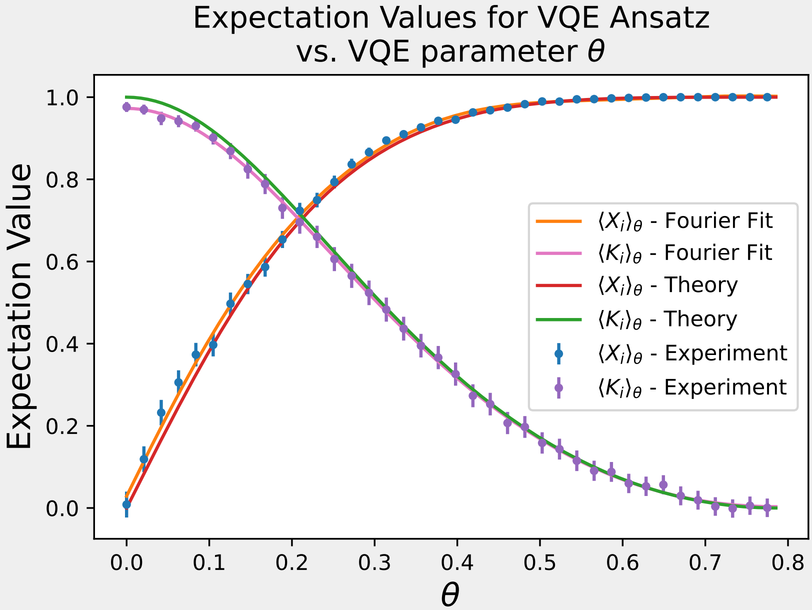
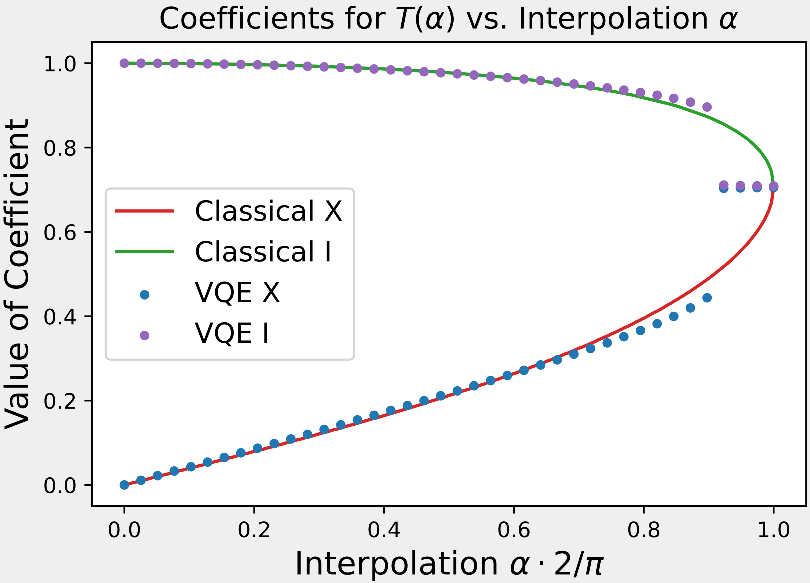
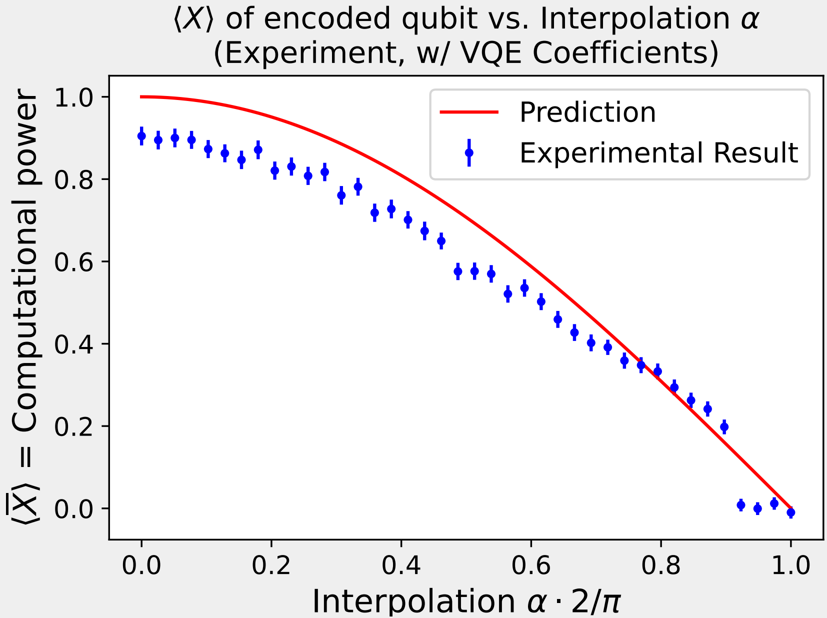


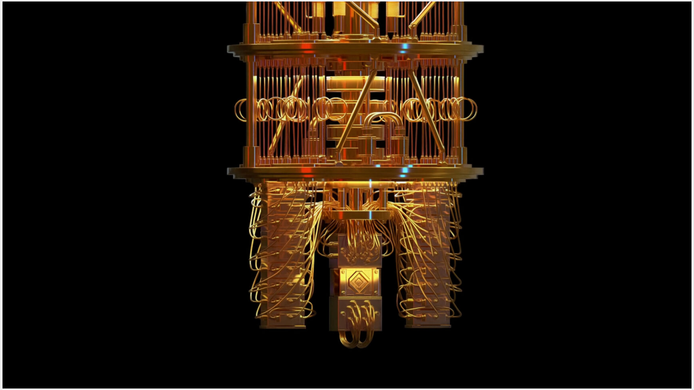
- Qubits disentangle... or are removed completely.
- Gain: Circuits of $2n$ qubits reduce to constant small sizes. Preparing the state is inefficient, but measuring the state (intriguingly) is not.


$\langle X_i \rangle_\theta$ and $\langle K_i \rangle_\theta$ for VQE Ansatz
vs. VQE parameter θ
Expectation Value
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
θ
$\langle X_i \rangle_\theta$
$\langle K_i \rangle_\theta$


Expectation Values for VQE Ansatz
vs. VQE parameter θ
Expectation Value
θ
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\langle X_i \rangle_\theta$ - Fourier Fit
$\langle K_i \rangle_\theta$ - Fourier Fit
$\langle X_i \rangle_\theta$ - Theory
$\langle K_i \rangle_\theta$ - Theory
$\langle X_i \rangle_\theta$ - Experiment
$\langle K_i \rangle_\theta$ - Experiment

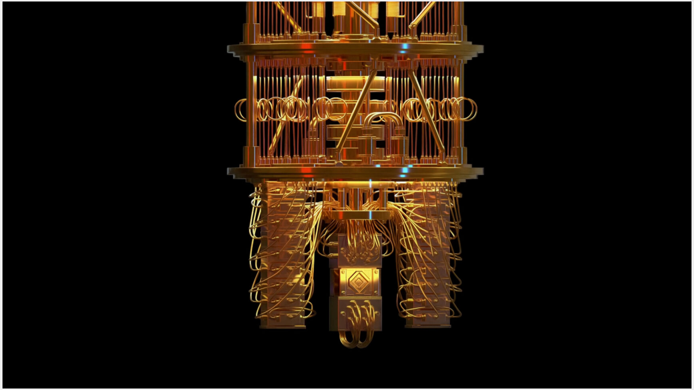

Coefficients for $T(\alpha)$ vs. Interpolation α
Value of Coefficient
Interpolation $\alpha \cdot 2/\pi$
Classical X
Classical I
VQE X
VQE I
0.0
0.2
0.4
0.6
0.8
1.0


⟨X⟩ of encoded qubit vs. Interpolation α
(Experiment, w/ VQE Coefficients)
Prediction
Experimental Result
⟨X̄⟩ = Computational power
Interpolation α · 2/π
0.0
0.2
0.4
0.6
0.8
1.0



- Shift to larger systems to demonstrate techniques for decoherence mitigation.

Image Credit: Quanta Magazine



- Shift to larger systems to demonstrate techniques for decoherence mitigation.
- Impact: First experimental demonstration of robustness of quantum computational power. Understanding quantum advantage, and a use case for NISQ devices.

Image Credit: Quanta Magazine