A Simulation of a Simulation: Algorithms for Measurement-Based Quantum Computing Experiments

APS NWS Meeting 2022

Rio Weil

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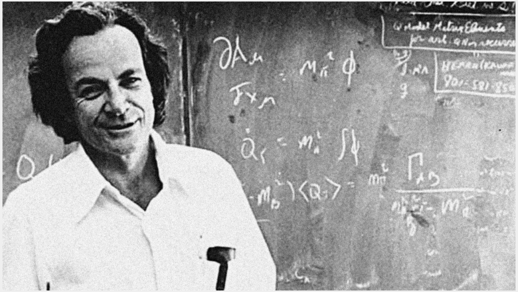
Outline

Motivation - What are Quantum Computers Good For?



“Nature isn’t classical, dammit, and if you want to make a simulation of nature, you’d better make it quantum mechanical, and by golly it’s a wonderful problem, because it doesn’t look so easy.”

- Feynman, 1981

Image Credit: Caltech Magazine

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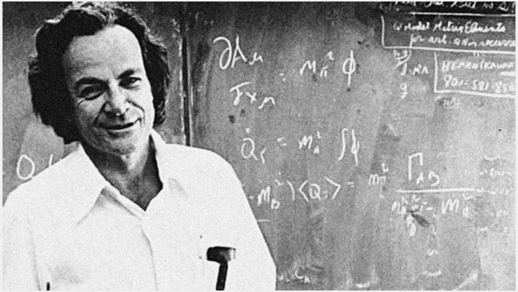

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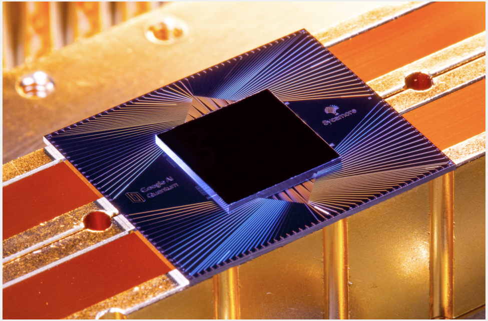
Also...

- Shor's Factoring Algorithm
- Grover's Search Algorithm

Image Credit: Caltech Magazine

Motivation - Outstanding Questions in Quantum Computing



1. What is the source of quantum advantage?

2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?

Image Credit: Erik Lucero/Google

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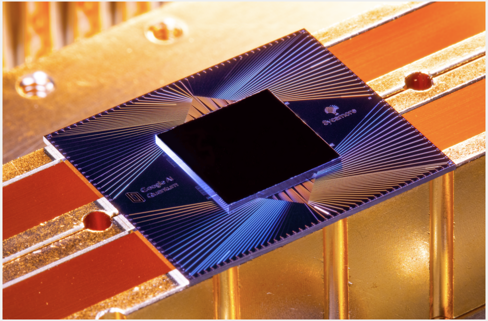
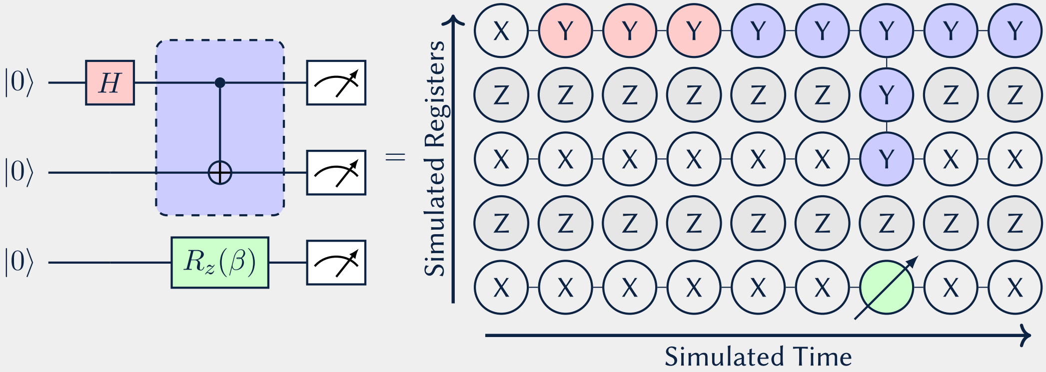


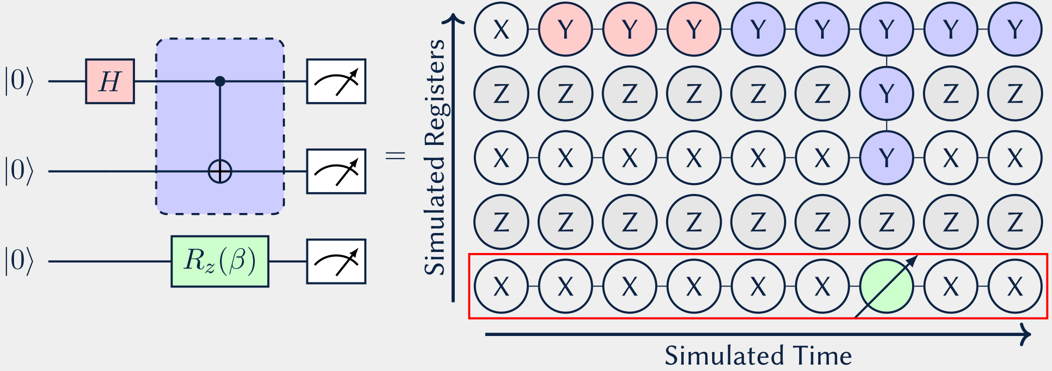
1. What is the source of quantum advantage?
   - Measurement-Based Quantum Computing - coming up!
2. What can we do with NISQ (Noisy Intermediate-Scale Quantum) devices?
   - Active research area... and this project!

Image Credit: Erik Lucero/Google



	Gate Model	MBQC
Evolution Method	Unitary Gates	Single-Qubit measurements
"Power Source"	Intermediate Gates	Initial State

Why Measurement-Based Quantum Computing?



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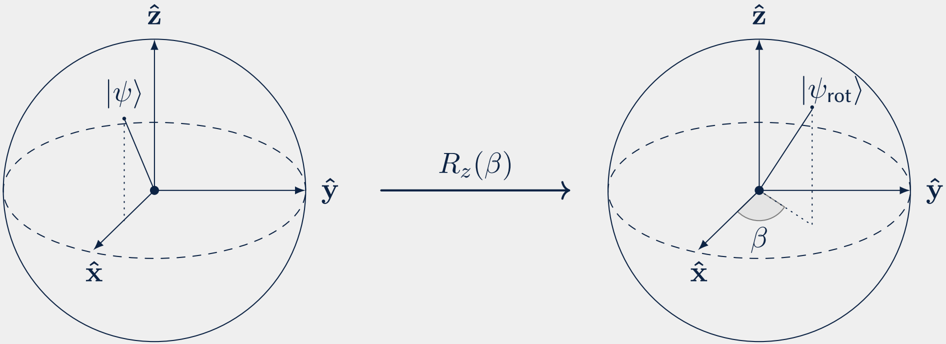
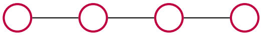


Ability to perform single qubit unitaries - rotations.

Universal Resource: Cluster State $|C\rangle$



Ground state of H_{cluster}

Useless Resource: Product State $|+\rangle^{\otimes N}$

Ground state of H_{product}

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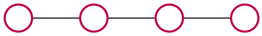

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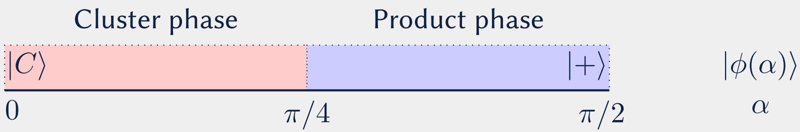
Ground state of H_{product}

Question: Power of ground states $|\phi(\alpha)\rangle$ of:

$$H(\alpha) = \cos(\alpha) H_{\text{cluster}} + \sin(\alpha) H_{\text{product}}?$$

1D Resource States - Phase Diagram & Decoherence

Answer (for infinite systems):



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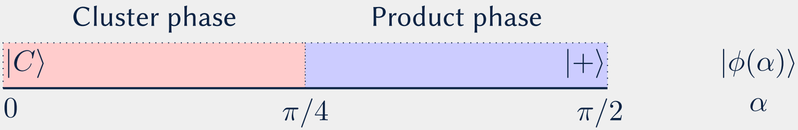

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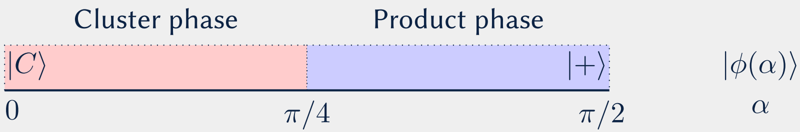
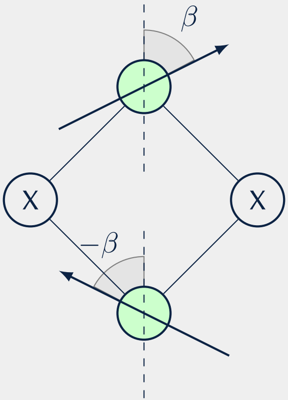
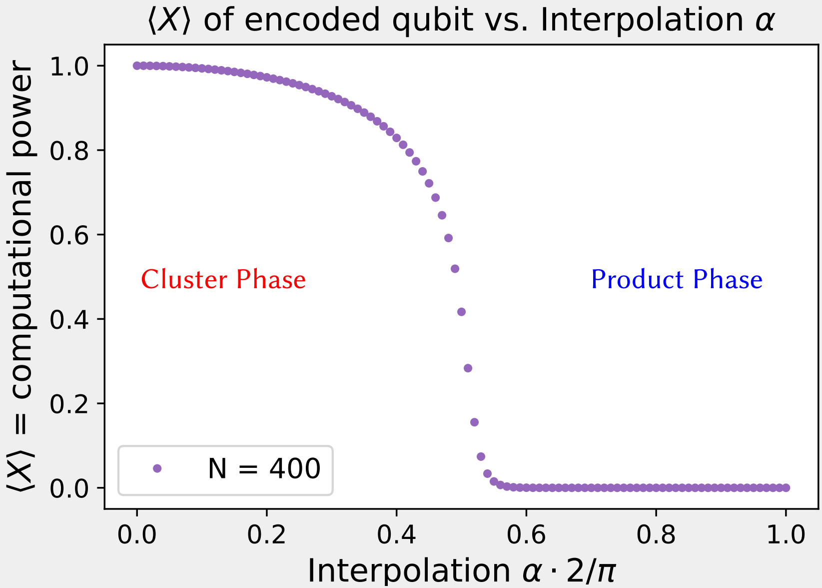


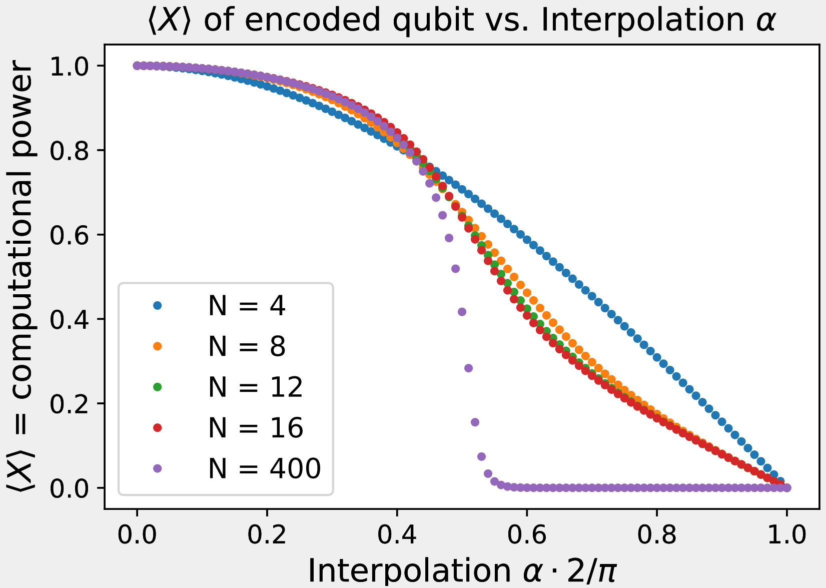
- The catch: Decoherence away from cluster state.
- Recently: New formalism for analyzing power in finite resource states.



Demonstration: The rotation-counter rotation scheme

1. Prepare $|\phi(\alpha)\rangle$, and input $|+\rangle$.
2. Apply β rotation, and $-\beta$ counterrotation via measurement.
3. Measure $\langle X\rangle$: computational power.


⟨X⟩ of encoded qubit vs. Interpolation α
⟨X⟩ = computational power
1.0
0.8
0.6
0.4
0.2
0.0
Cluster Phase
Product Phase
N = 400
0.0
0.2
0.4
0.6
0.8
1.0
Interpolation α · 2/π



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Interpolation α · 2/π
1.0
0.8
0.6
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0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
N = 4
N = 8
N = 12
N = 16
N = 400

From Theory to Experiment - Producing Ground States

Recall $H(\alpha)$ and corresponding ground state $|\phi(\alpha)\rangle$:

$$H(\alpha) = \cos(\alpha) H_{\text{cluster}} + \sin(\alpha) H_{\text{product}}$$

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Further, we can exchange unitarity for a simpler representation:

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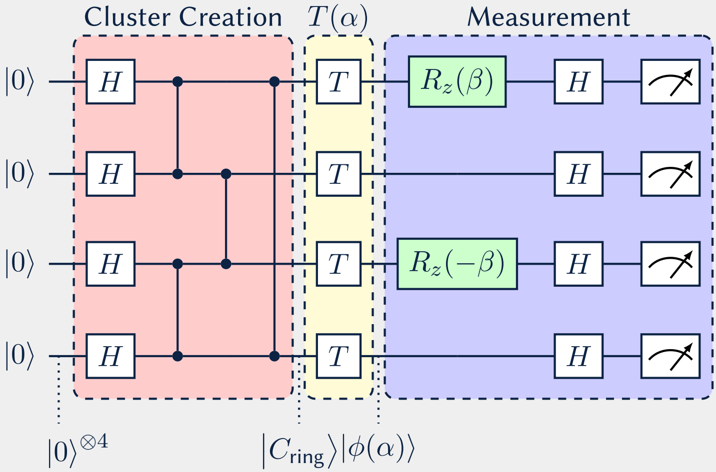
Finally, we can assume T is local, so:

$$T(\alpha) = \bigotimes_{i=1}^{N} (aX_i + bI_i)$$

With these simplifications, $T(\alpha)$ can be found via classical optimization, for small rings.


Cluster Creation
$T(\alpha)$
Measurement
$|0\rangle$
$|0\rangle$
$|0\rangle$
$|0\rangle$
H
T
$R_z(\beta)$
$R_z(-\beta)$
$|0\rangle^{\otimes 4}$
$|C_{\text{ring}}\rangle|\phi(\alpha)\rangle$

Problem: $T(\alpha)$ is non-unitary in general. But quantum gates are unitary.

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$$p(\mathbf{j}) = |\langle \mathbf{j}|\phi(\alpha)\rangle|^2 = |\langle \mathbf{j}| [T(\alpha)|C\rangle]|^2 = |[\langle \mathbf{j}| T(\alpha)]|C\rangle|^2 = \left|\left\langle T^\dagger(\alpha)\mathbf{j}\middle|C\right\rangle\right|^2$$

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- Conclusion: Don't implement $T(\alpha)$ at all. Instead, measure $T^\dagger(\alpha) |\mathbf{j}\rangle$ on $|C\rangle$ instead!

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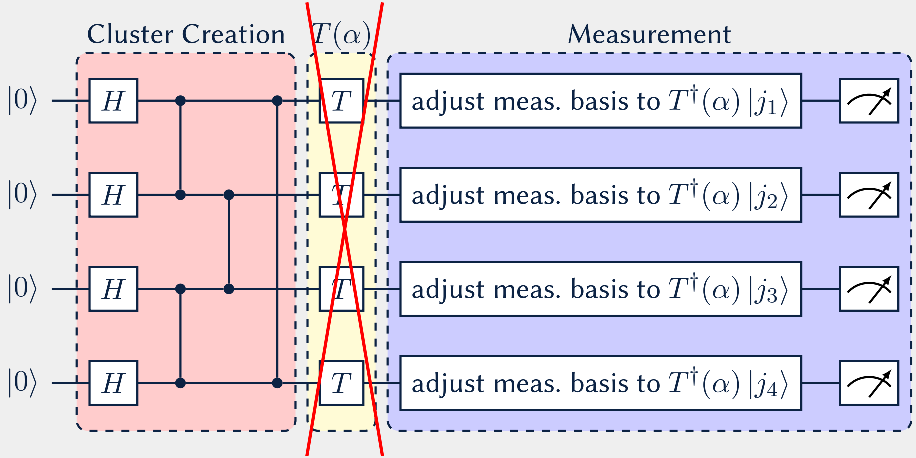
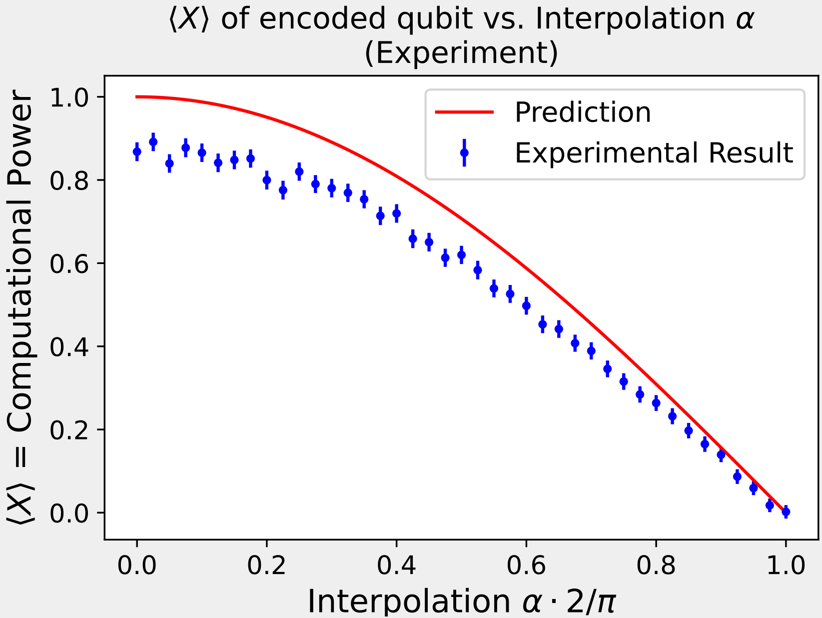
- Takeaway: Problem Decomposition.



The Task: Develop algorithm to make circuits and post-process measurement outcomes, obtain results *as if* $T(\alpha)$ had been implemented.



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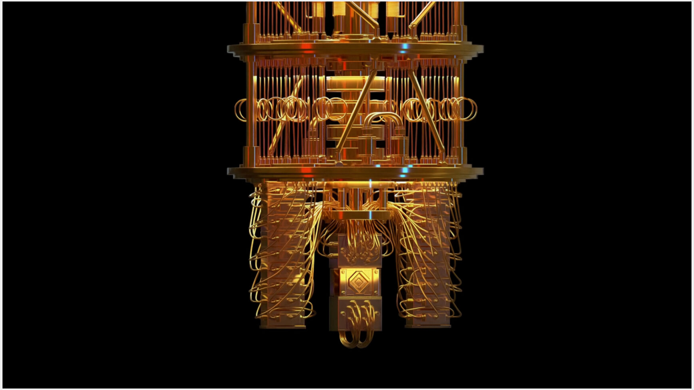
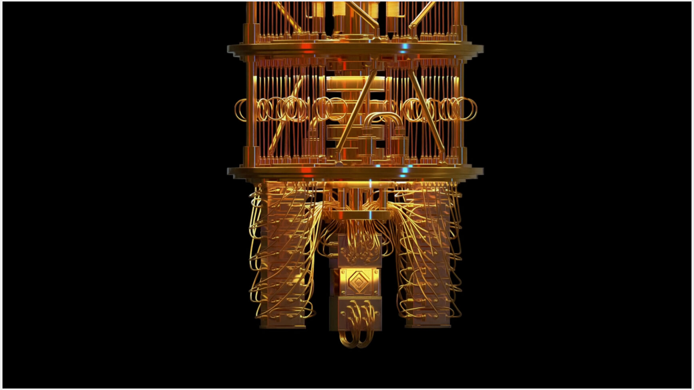


Image Credit: Quanta Magazine



- To-Do: Larger systems to demonstrate decoherence mitigation.

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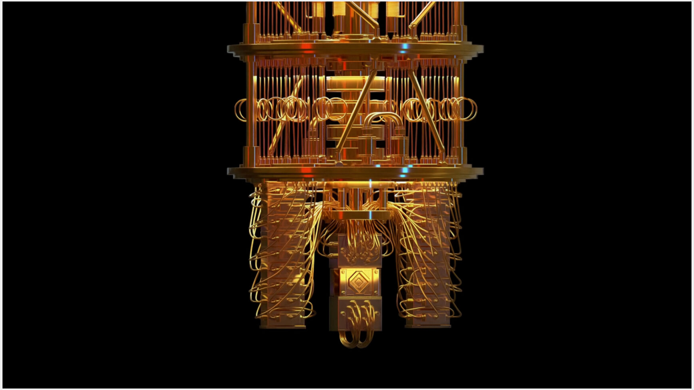
Outlook & Conclusion



- To-Do: Larger systems to demonstrate decoherence mitigation.
- Impact: First experimental demonstration of robustness of quantum computational power. Understanding quantum advantage, and a use case for NISQ devices.

Image Credit: Quanta Magazine