PHYS 143 Practice Midterm

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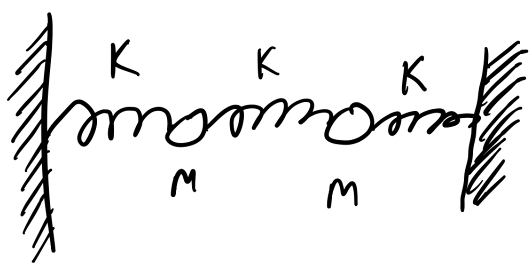
Time: 80 minutes(?) (These problems are longer than what I think would be reasonable to complete in the 50 minute time that is your actual midterm - still good to try to write this in a timed/pressured setting, though!)

1. /50
2. /50

1 Problem 1 (Normal modes and damped/driven oscillations) (50 pts)



(a) (17.5pts) Consider the above system of two masses of mass m connected by springs of spring constant k. Write down the matrix differential equation for their motion using Newton's law. Find the normal frequencies and modes, and describe the normal mode motion. For this part, neglect damping.

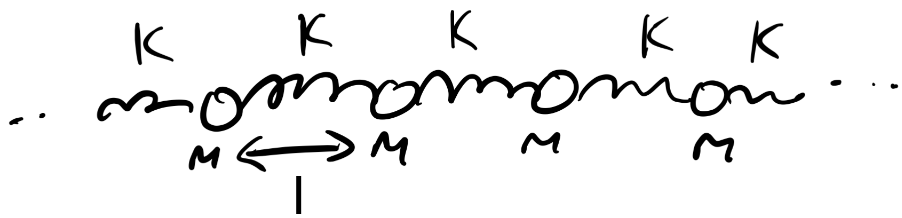
(b) (5pts) Suppose that we now add weak linear damping to both masses (e.g. as air drag $\mathbf{F}_{\text{drag}} = -b\mathbf{v}$). Without doing a calculation - how would you expect the two normal frequencies you found in (a) to change?

(c) (7.5pts) What kind of damping (over/under/critical) do the updated frequencies in (b) correspond to? Sketch what the motion of the left mass would look like for each of the two new normal modes.

(d) (15pts) For this part again suppose there is no damping. Suppose we drive the left mass with driving force $F = F_d \cos(\omega t)$. Look for a particular solution of the form $\mathbf{x} = \mathbf{A}\cos(\omega t)$ to obtain for the amplitudes A_L, A_R of the two masses (hint work in normal coordinates $x_L + x_R$ and $x_L - x_R$ and find the amplitudes of the normal modes, then use this to find the amplitudes of the individual masses).

(e) (5pts) What resonance frequency do you find in (d), and what happens at resonance? Your answer may be unphysical - what would be different if you actually drove the system at resonance in reality?

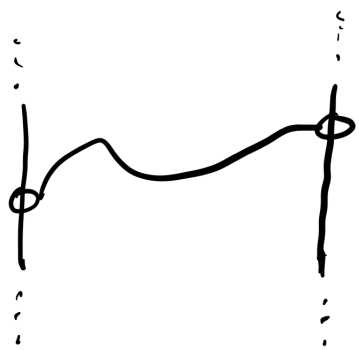
2 Problem 2 (Wave equation) (50pts)



(a) (5pts) Consider a chain of N masses m connected by springs k of equilibrium spacing l and total length L. Use Newton's Laws to write down the equation of motion for a given single mass in the chain, using $\psi(x,t)$ to denote the distance from equilibrium of the mass at x.

(b) (5pts) Derive the wave equation $\partial_x^2\psi(x,t) = \frac{1}{c^2}\partial_t^2\psi(x,t)$ from this setup by taking the continuum ($N \to \infty$, $l \to 0$) limit. What is the wave speed of a wave c that propagates in this system? (Hint: You can check that your answer makes sense via dimensional analysis).

(c) (5pts) What is the dispersion relation of the frequency ω of waves and the wavenumber k for this system? (Hint: Guess a solution of the form $\psi(x,t) = Ae^{ikx-\omega t}$).



(d) (15pts) Suppose that both ends of the string are free to move, so the string has boundary conditions $\psi'(0) = \psi'(L) = 0$. Check that solutions of the form $\psi_n(x,t) = \cos(k_n x)e^{\pm i\omega_n t}$ satisfy the boundary condition with $k_n = \frac{n\pi}{L}$ for $n = 0, 1, \ldots$. Given this, write down the most general solution to the wave equation that satisfies these boundary conditions. Sketch the standing waves for $n = 0, 1, 2$.

(e) (15pts) Now, suppose that $\psi(x, t = 0)$ is given by the square wave:

$$\psi(x, t=0) = \begin{cases} A & 0 \le x \le L/4 \\ 0 & L/4 < x < 3L/4 \\ A & 3L/4 \le x \le L \end{cases}$$

and that $\partial_t\psi(x, t = 0) = 0$. Find the exact solution to the wave equation that satisfies these initial conditions, starting from the general solution you found in (d).