PHYS 141 Discussion/Tutorial 1

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1 Do Love Quadrangles Lead to Logarithmic Spirals?

A, B, C, D are four friends in a rom-com film that are unfortunately stuck in a love quadrangle (A likes B likes C likes D likes A). They start their day at their houses, located on the four corners of a square, inscribed on a circle of radius R.

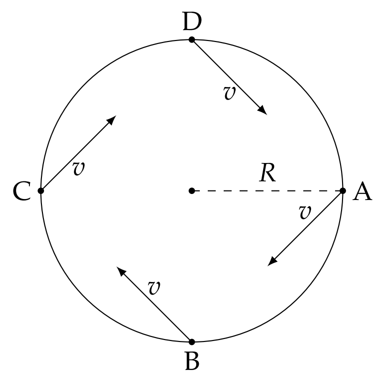

Figure 1.1: Friends $ABCD$ are initially located on four corners of a square, inscribed on a circle of radius R. At all times, each friend walks towards their direction of their romantic interest with speed v; shown are the velocity vectors at time $t = 0$.

Each of them pursues their love interest at a constant speed v. In this problem, we will study their trajectories in time.

(a) In physics, we often have difficult seeming problems that can greatly be simplified in the presence of symmetries. Let's think about how symmetries can help us with this problem.

  (i) At $t = 0$, $ABCD$ form a square. What does their shape look like at later times?

  (ii) At some time T later, $ABCD$ will eventually reach each other; what point would we expect this to be?

  (iii) What coordinate system would be convenient for analyzing this problem?

(b) Since each of the trajectories of the four friends are the same up to a rotation, let's analyze the trajectory of A. In (a), you should have found that polar coordinates is a convenient coordinate system for analyzing this problem. What is As velocity in polar coordinates?

(c) Using your result in (b), at what time T do friends $ABCD$ meet each other?

(d) Write down the general form of the velocity in polar coordinates. Using this, solve for the radius as a function of time $r(t)$. Then, solve for the angle as a function of time $\phi(t)$.

(e) Solve for $r(\phi)$; can you tell why their trajectory is often called a "logarithmic spiral?"

(f) In HW1, you looked at the arc length:

$$L = \int dL = \int \sqrt{dx^2 + dy^2} = \int \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} dt \tag{1.1}$$

for a ball thrown at angle θ. In polar coordinates (i.e. in terms of r, ϕ) what does the length element $dL^2 = dx^2 + dy^2$ become? Using this, write down the integral for the arc length and solve it. You can also find the answer without using any calculus whatsoever; thinking about the problem geometrically (Hint: each of the friends travel at right angles at all times). Do the two methods agree?

(g) (BONUS) **Self-Similar Spirals.** A path $r = f(\phi)$ in polar coordinates is *self-similar* (i.e. a fractal) if zooming into the curve only results in a shift of the angle, i.e. scaling r by α results in a shift by the angle by c:

$$\alpha r = f(\phi - c) \tag{1.2}$$

Take the derivative of Eq. (1.2) by both sides w.r.t. α, and then set $\alpha = 0$ to obtain the differential equation:

$$f(\phi) = -f'(\phi)c'(0). \tag{1.3}$$

Finally, solve this differential equation, and check that the solution coincides with your answer to (e); this tells us that the paths that the friends take are self-similar/fractals!

(h) (BONUS) **Love N-angle.** How do your answers change for (b)-(f) if we instead have N friends who start on the edges of an N-gon? $N = 2, 4$ are good checks to see that your answer makes sense.