PHYS 142 (Honours Intro E&M) Practice Final

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Try all 8 problems. You must show all of your work for full points. Time limit: 120 minutes.

1. /10
2. /10
3. /10
4. /10
5. /10
6. /15
7. /10

Question 1

For each question, answer whether the statement is true or false, and explain why (2.5pts each)

(a) $\mathbf{E} = Ay\hat{\mathbf{x}} + Ax\hat{\mathbf{y}} + Az\hat{\mathbf{z}}$ describes an allowed static electric field with a constant charge density throughout space.

(b) In a universe where the electric field distance dependence of a point charge was $\sim \frac{1}{r}$, the far-field distance dependence (that is, for $r \gg l$ with l the charge separation) of a dipole would be $\sim \frac{1}{r^2}$.

(c) There is no image charge configuration that could be used to find the force on a proton that is halfway between two grounded conducting plates.

(d) In the absence of a current $\mathbf{J}$, both the divergence and curl of the magnetic field must vanish.

Question 2

Suppose we have a slab of uniform volume charge density ρ, that extends infinitely in the x/y directions and from $z = d$ to $z = -d$. We then carve out a spherical hole of radius $r = a < d$ from the slab. Find the electric field along the $\hat{\mathbf{z}}$ axis (magnitude and direction), and plot the electric field strength as a function of z for $z \geq 0$.

Question 3

Consider a parallel plate capacitor with charge $\pm Q$ and area A, with plate separation d.

(a) Derive the capacitance. (4pts)

(b) Calculate the energy stored in the capacitor by considering the energy it would take it to charge it up from 0 charge. (3pts)

(c) Calculate the the energy stored in the capacitor by considering the energy it would take to pull the $\pm Q$ charged plates together from distance 0 to d. Your answer should agree with (b). (3pts)

Question 4

Suppose we have a infinite line (extending from $x = \infty$ to $x = -\infty$ with $y = z = 0$) of uniform linear charge density λ. We now consider a charge $+q$ a distance d away from the line, travelling with $\mathbf{v} = v\hat{\mathbf{x}}$.

(a) Compute the electric field and force felt by the charge in the stationary frame. (2pt)

(b) Suppose we now enter the frame of the charge. How does the force it feel change, and why? How does the electric field felt at the point of the charge change, and why? Based on this, why would you argue that there must exist a magnetic field? (5pt)

(c) Calculate the electric and magnetic fields in the charge/moving frame. Check that the force on the charge agrees with what you argued in (b) (3pt)

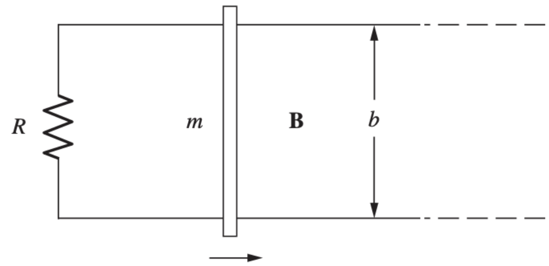
Question 5

Consider an infinite sheet of uniform surface current $\mathbf{K} = K\hat{\mathbf{x}}$ flowing over the xy plane.

(a) Calculate the magnetic field everywhere. (6pts)

(b) Find an expression for the vector potential $\mathbf{A}$ everywhere. Is your solution unique? (4pts)

Question 6



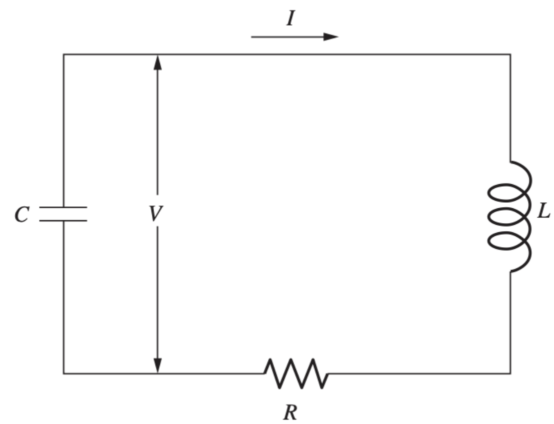
As is shown above, two perfectly conducting rails separated by distance b sit in a uniform magnetic field of strength B pointing into the page. The rails are connected at one end by a fixed resistor of resistance R, and a freely movable bar with mass M, with initial velocity $\mathbf{v} = v_0\hat{\mathbf{x}}$.

(a) Even though there is no friction, the bar is observed to slow down; without any calculation, explain why. (3pts)

(b) If the bar is initially located at $x = 0$, determine the time when the bar stops moving and the final location of the bar (7pts)

(c) Show explicitly where the initial kinetic energy of the bar goes (5pts).

Question 7



Consider the above LRC circuit.

1. Derive a differential equation for the potential V. In analogy with Newton's equations of motion, explain briefly the effect of each of the inductor/resistor/capacitor on the potential. (5pts)
2. Assume a solution of the form $V(t) = Ae^{-\alpha t}e^{i\omega t}$, and derive α, ω in terms of R, L, C. Sketch the solution. (5pts).